9-1-2012

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Timothy P. Hubbard

Justin Svec
College of the Holy Cross, jsvec@holycross.edu

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COLLEGE OF THE HOLY CROSS, DEPARTMENT OF ECONOMICS
FACULTY RESEARCH SERIES, PAPER NO. 12-02

Department of Economics
College of the Holy Cross
Box 45A
Worcester, Massachusetts 01610
(508) 793-3362 (phone)
(508) 793-3708 (fax)

http://www.holycross.edu/departments/economics/website

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A Model of Tradeable Capital Tax Permits

By

Timothy P. Hubbard†
Colby College

and

Justin Svec‡
College of the Holy Cross

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Abstract

Standard models of horizontal strategic capital tax competition predict that, in a Nash equilibrium, tax rates are inefficiently low due to externalities - capital inflow to one state corresponds to capital outflow for another state. Researchers often suggest that the federal government impose Pigouvian taxes to correct for these effects and achieve efficiency. We propose an alternative incentive-based regulation: tradeable capital tax permits. Under this system, the federal government would require a state to hold a permit if it wanted to reduce its capital income tax rate from some pre-determined benchmark. These permits would be tradeable across states. We show that, if the federal government sets the correct number of total permits, then social efficiency is achieved. We discuss the advantages of this system relative to the canonical suggestion of Pigouvian taxes.

JEL Classification Codes: H25, H42, H70

Keywords: tax competition; marketable permits; asymmetric states

We thank Gagan Ghosh, B. Ravikumar, and David E. Wildasin for helpful comments. Of course, any errors are our own.

††Department of Economics, Colby College, 5242 Mayower Hill, Waterville, ME 04901 207-859-5242 (phone), 207-859-5248 (fax), timothy.hubbard@colby.edu

†††Department of Economics, Box 45A, College of the Holy Cross, Worcester, MA 01610-2395, 508-793-3875 (phone), 508-793-3708 (fax), jsvec@holycross.edu
1 Introduction

One of the key insights in the tax competition literature is that policy competition across states leads to socially inefficient policy choices. The inefficiency is due to the presence of externalities, which can be particularly salient when considering a state’s tax rate on a relatively mobile factor like capital. Specifically, in its competition for scarce capital, each state has the incentive to cut its capital income tax rate. This reduction spurs an inflow of capital, raising wages, total tax revenues, and public-good provision. A capital inflow towards one state, though, means a capital outflow from other states. This outflow implies that other states will confront lower wages, a decreased tax base, and hence lower public-good provision. These welfare costs to other states are not internalized by the state considering a decrease in its capital income tax rate. Consequently, from a social perspective, each state sets an inefficiently low tax on capital income.

The responses of other states compound the problem: the other states respond by decreasing their own capital income tax rate, hoping to prevent the capital outflow. As such, the tax competition literature predicts a “race to the bottom” in which all states feel obliged to set suboptimally low tax rates on capital. One implication of this is that states must either increase other taxes (e.g., labor, consumption, property, etc.) to supplement lost capital tax revenue or decrease their provision of public goods.

A recent example of this incentive to poach other states’ capital involves Illinois and Wisconsin. Due to its large debt, Illinois was recently forced to raise its tax rates on both personal and corporate income. The Wisconsin governor, Scott Walker, responded to this move, saying “Today we renew that call to Illinois businesses, ‘Escape to Wisconsin.’ You are welcome here.”¹ In support of this sentiment, the Wisconsin State Legislature passed a proposal that would eliminate the corporate income tax rate for two years for firms that relocate to Wisconsin from other states.²

In addition to this anecdote, there is evidence of tax competition at all levels of government. Slemrod (2004) noted that corporate tax rates across countries began declining in the 1980s and seemed to be converging, facts that are consistent with tax competition. While his goal was

to try and disentangle competing arguments for whether this observation is, in fact, evidence of tax competition, he found measures of openness are negatively associated with corporate tax rates. Likewise, Devereux, Lockwood, and Redoano (2008) found evidence of strategic interaction between countries with open economies and concluded that reductions in tax rates can be explained almost entirely by more intense competition. Carlsen, Langset, and Rattsø (2005) used data from Norway to construct a measure of firm mobility which they used to show that municipalities with high firm mobility tend to have lower tax levels. Davies (2005) provided evidence that U.S. states compete in offering incentives, often in the form of firm-specific tax reductions, to multinational enterprises in trying to attract foreign direct investment, which likely transfer rents from the states to the firms. Thus, the incentive for states to attract capital from other states by offering low capital tax rates seems both theoretically and empirically relevant.

The incentives driving one state to reduce its capital income tax rate in order to attract other states’ capital (or protect the loss of its own capital) can be costly in terms of national welfare. In an international model with competing countries, Mendoza and Tesar (1998) predicted that if the U.S. eliminated its capital income tax rate, then Europe would face a decrease in welfare that is comparable to a 1.7% decline in trend consumption. This large welfare cost is due to the fact that Europe not only faces a capital outflow, but also must increase its tax on consumption in order to maintain fiscal solvency.

Many researchers have proposed ways to solve the tax competition problem. These solutions fall into two categories. The first type of solution calls on all states to jointly increase their tax rates; see, for example, Batina (2009). This suggestion would give the states the chance to escape the welfare-reducing Nash equilibrium. The difficulty, of course, is that the agreement is not incentive compatible—each state would have the incentive to cheat on the agreement by reducing its capital income tax to attract other states’ capital. Because of this, the agreement to jointly increase tax rates can only be supported with costly monitoring and enforcement.

The second type of solution proposes that the federal government impose a (state-specific) Pigouvian tax on states that reduce their capital tax rates; see, for example, Wildasin (1989) as

\[\text{3For similarly-sized states, this game can be thought of as an example of the canonical prisoner’s dilemma or tragedy of the commons games.}\]
well as DePeter and Myers (1994). If the taxes are set correctly, the benefit of this approach is that each state fully internalizes the social cost of modifying its tax rate. Consequently, all states choose the socially efficient level of capital taxes. A disadvantage of this second option is informational: it is difficult for the federal government to acquire the requisite information needed to determine the efficient values of the Pigouvian tax for each state. This problem is made more severe by the fact that each state has the incentive to deceive the federal government about its characteristics in the hope of influencing its Pigouvian tax.  

In this paper, we propose an alternative solution which, like the Pigouvian tax, induces the states to internalize the externality associated with their actions, while simultaneously reducing the informational burden imposed on the federal government. Specifically, we consider establishing a market for capital income tax reduction permits, where a permit is defined as the right to reduce a state’s capital income tax rate from a benchmark value by one percentage point for some specified period of time.  

With this market in place, the number of permits a state holds determines how much the state can decrease its capital income tax rate from its benchmark. A public exchange would allow states that wanted to lower their capital tax rate to purchase permits from other states, whose capital tax rates would necessarily rise in response. Importantly, we show that if the total number of permits is set correctly, the market price of the permits reflects the size of the externality associated with reducing an individual state’s capital income tax rate. Each state then fully internalizes the social cost of its action since purchasing (holding) a permit is costly (involves an opportunity cost). As such, the permit system induces states to choose to implement the socially efficient capital income tax rates.

While our application is new, the benefits of tradeable permits have been well studied in other settings within the fields of environmental economics and auctions. We borrow insights from the

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4 A recent example of this behavior in a different context is the recent budget crisis in Greece—the country was accused of trying to hide its excessively high debt level. In fact, an audit of the Greek budgetary statistics by Eurostat (2004) suggested that the country was under-reporting its budget deficit, noting that an upward revision of the country’s deficit situation was “exceptional” and gave rise to “questions about the reliability of the Greek statistics on public finances.”

5 We have decided to apply this market solution to capital income tax rates, but there is no reason why the idea should be limited to just this type of tax competition. Indeed, the idea could apply to any fiscal instrument that causes an externality in another state, including the labor income tax rate and spending on public assistance programs.
environmental economics literature where permits represent the right to emit a unit of pollution. The establishment of the U.S. market for tradeable sulfur dioxide (SO$_2$) emissions permits in Title IV of the 1990 Clean Air Act Amendments was considered the first large-scale environmental program to rely on private markets. The main features of the program and early documentations of the program’s success were described by Joskow, Schmalensee, and Bailey (1998), among others. Joskow et al. argued that a relatively efficient market price for permits was achieved after just a few years—something that is quite remarkable when taking into consideration the number of facilities that emit SO$_2$ in the country. In contrast, the setting we study is simpler in that the players (decision makers) are clearly defined to be the states’ governing bodies. Such transparency could allow for efficiency even quicker.

Our application is perhaps closest in spirit to that of Casella (1999) who proposed the use of marketable permits as a way of achieving deficit spending goals of the Pact for Stability and Growth (Stability Pact) in the European Monetary Union (EMU). The Stability Pact suggested a ceiling for each member country’s deficit spending of 3% of its GDP. Casella emphasized practical disadvantages and weaknesses of the proposed deficit ceiling and suggested how they could be addressed in a more flexible system of tradeable deficit permits. A virtue of her suggestion is that it not only aligns countries’ incentives with those of the EMU, but also that it allows country-specific idiosyncratic shocks to be smoothed by the acquisition or sale of permits. This feature holds in our application as well if public goods expenditures can substitute for fluctuations in other areas of the economic system.

Our paper is structured as follows: in Section 2, we present a model in which potentially asymmetric states compete for scarce capital by adjusting their capital income tax rates. We derive the optimal tax rates that emerge when states act competitively and when a social planner can choose all states’ tax rates. Given these equilibria, we document the inefficiency that occurs when states set tax policy competitively. These results, and the underlying model, follow Bucovetsky (1991). In Section 3, we propose our novel solution to the inefficiency created by tax competition. Upon formalizing the market for permits, we show that each state has the incentive to choose the socially efficient capital tax rate, as long as the cap on total permits is set correctly. We also show
that, even if the aggregate number of permits is set inefficiently, the permits system preserves some attractive social welfare features. We conclude in Section 4.

2 Model

Consider a model of capital tax competition as presented by Bucovetsky (1991) and Wilson (1991). Specifically, a country is divided into two, possibly asymmetric, states. In each state, capital and labor are used to produce a single, homogenous good according to the per-capita production function \( f(k_i) \), where \( k_i \) is the per-capita capital stock in state \( i \in \{1, 2\} \). The homogenous good’s price is set to one. Assume the production function is concave in \( k_i \), twice continuously differentiable, and exhibits constant returns-to-scale in capital and labor. The factor markets are perfectly competitive. Let \( w_i \) and \( r_i \) be the wage rate and the net return on capital, respectively, in state \( i \). The gross return on capital in state \( i \) is then \( f'(k_i) \), and the wage is

\[
    w_i = f(k_i) - f'(k_i) k_i.
\]

Likewise, the net return on capital is

\[
    r_i = f'(k_i) - t_i
\]

where \( t_i \) corresponds to the capital income tax rate in state \( i \). We assume that state \( i \)'s tax rate is such that \( r_i > 0 \).

We consider capital to be perfectly mobile across states, while labor is immobile. The former assumption implies that, in equilibrium, the after-tax rate of return on capital is equal across states:

\[
    r \equiv f'(k_1) - t_1 = f'(k_2) - t_2. \tag{1}
\]

Further, the nation’s capital-labor ratio is fixed at \( k^* \). It can be shown that \( k^* \) is equal to a weighted

\footnote{Bucovetsky (1991) provided a detailed characterization of the “excess supply” regime in which \( r_i = 0 \). His results apply to our setting too, although we do not present them formally as the supply of capital in one state is independent of that of other states in such cases and so the incentive effects we seek to highlight are no longer relevant. Moreover, Bucovetsky showed that, under fairly mild assumptions, equilibria with excess supply of capital can be ruled out—see Lemmata 3 and 4 of his paper. Wilson (1991) also choose to neglect the “empirically irrelevant” case where \( r_i = 0 \).}
average of each state’s capital-labor ratio:

\[ k^* = s_1 k_1 + s_2 k_2 \]  
(2)

where \( s_i = \frac{L_i}{L_1 + L_2} \) represents the population share of state \( i \).

Output can be consumed either as a private good, \( x \), or as a public good, \( g \). Consumers derive utility from both of these goods. We assume that the utility function is quasi-linear in private and public consumption

\[ u(x_i, g_i) = v(x_i) + g_i. \]

This assumption, while limiting, is necessary for the proof of theorem 1 in Section 3. Consumers purchase the private good using their income, which is composed of labor and capital income. Assuming that each consumer inelastically supplies one unit of labor, a consumer’s labor income is equal to the wage. We also assume that each consumer holds an equal share of the nation’s capital stock so that a representative consumer’s capital income is then \( r k^* \). As there is no saving in the model, the consumers spend all of their income on the private good:

\[ x_i = w_i + r k^* = f(k_i) - f'(k_i) k_i + r k^* \]  
(3)

where we make implicit use of the fact that the output good is the numéraire good.

The provision of the public good is funded by each state government’s capital tax revenues. Given that state \( i \)’s tax \( t_i \) applies to all capital employed in state \( i \), tax revenues are \( t_i k_i \). Consequently, each government’s budget constraint is

\[ g_i = t_i k_i. \]  
(4)

We assume that each state government is benevolent and seeks to maximize the utility of the consumers who live in that state.
2.1 The Competitive Equilibrium

Suppose that both state governments set their capital tax rates in a non-cooperative fashion, taking the other state’s tax as given. The optimization problem for state $i$’s government, then, is

$$\max_{t_i} u(x_i, g_i)$$

subject to (1)–(4).

To simplify this problem, note that for a given pair of tax rates $(t_i, t_j)$, the capital-labor ratio in each state is determined by (1) and (2). Call the resulting capital-labor ratio $k_i(t_i, t_j)$. We can use this result to analyze how the capital-labor ratio moves with respect to the tax rate. Given that the national capital-labor ratio is fixed, it is straightforward to show that

$$\frac{\partial k_i}{\partial t_i} = \frac{s_j}{s_j f''(k_i) + s_i f''(k_j)} < 0$$

where the inequality holds because of the concavity of the production function. This equation suggests that there is a capital inflow (outflow) when state $i$ lowers (raises) its tax rate. Furthermore, the sensitivity of capital to the tax rate depends on the relative size of the state: as the population share of a state rises, the capital-labor ratio in that state becomes increasingly insensitive to tax changes. This suggests, as in Bucovetsky (1991), that large states face less incentive to reduce their tax rates in order to attract capital than do small states, a characteristic that will become important in the permits model below.

Using the fact that the capital-labor ratio can be written as a function of the pair of tax rates, we can also solve for the wage, the after-tax rate of return on capital, private good consumption, and public good consumption as functions of the tax rates. Call these values $w_i(t_i, t_j)$, $r_i(t_i, t_j)$, $x_i(t_i, t_j)$, and $g_i(t_i, t_j)$, respectively. Since $w_i(t_i, t_j)$ and $r(t_i, t_j)$ both fall with an increase in $t_i$,

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$^7$Non-cooperative solutions were considered for commodity taxes by Mintz and Tulkens (1986) as well as de Crombrugghe and Tulkens (1990). Wildasin (1988) compared equilibria of two types of seemingly similar fiscal competition games: one in which governments choose the tax rate on mobile capital and one in which governments choose a level of public expenditures. He found the two equilibria, in general, do not coincide even in symmetric models (where states or jurisdictions are identical) and that equilibria in expenditures entail more intense rivalry and result in lower levels of public good provision; thus, larger inefficiencies obtain in such settings.
we know that \( \frac{\partial x_i}{\partial t_i} < 0 \).

With these findings, we can now more fully understand the externality associated with capital tax competition. The government in state \( i \) has the incentive to reduce its tax rate in order to attract capital. This action increases the wage and the private good consumption of that state’s consumers. However, the decrease in the tax rate induces an outflow of capital from state \( j \) which reduces the wage and private good consumption of that state. When state \( i \) makes the decision to lower its tax rate, it does not internalize the harm that comes to state \( j \)’s consumers. As a consequence, state \( i \) chooses an inefficiently low capital tax rate.

The optimal, non-cooperative tax rate set by the government in state \( i \) solves the following optimization problem:

\[
\max_{t_i} v [x_i (t_i, t_j)] + g_i (t_i, t_j)
\]
subject to (1)–(4). The first-order condition of this maximization problem is

\[
v_x (x_i) \frac{\partial x_i}{\partial t_i} + \frac{\partial g_i}{\partial t_i} = 0
\]

(5)

where

\[
\frac{\partial x_i}{\partial t_i} = f''(k_i) \frac{\partial k_i}{\partial t_i} [k^* - k_i] - k^*
\]

and

\[
\frac{\partial g_i}{\partial t_i} = k_i + t_i \frac{\partial k_i}{\partial t_i} = k_i \left( 1 + \frac{\partial k_i}{\partial t_i} \frac{t_i}{k_i} \right).
\]

Note that the sign of \( \frac{\partial g_i}{\partial t_i} \) depends on the elasticity of the capital inflow to a state with respect to the state’s tax rate. By combining both states’ first order conditions, we can derive the Nash equilibrium pair of tax rates, \((t^*_1, t^*_2)\).

To gain insight about these tax rates, we numerically solve the model under the following assumptions: the utility and production functions are defined by

\[
u(x, g) = \log x + g
\]
and

\[ f(k) = (a - bk)k \]

respectively, where \( a = \frac{3}{2} \) and \( b = \frac{1}{3} \). The assumption of a quadratic production function is not random—this example is well rooted in the literature, having been leveraged extensively by Bucovetsky (1991) and employed by Wilson (1991) as well as Wildasin (1991).\(^8\) Assume initially a symmetric model in which \( L_1 = L_2 = 1000 \), and \( k^* = 1 \). These values are chosen merely for illustrative purposes and will be adjusted later in the paper. Given these assumptions, Figure 1 plots the marginal cost and benefit of state 1 modifying its capital tax rate, assuming that state 2 chooses \( t_2^* \) as its tax rate. The marginal cost of raising the tax is that capital will flood out of state 1, reducing private consumption. The value of this loss is \( v_x(x_i) \frac{\partial x_i}{\partial t_i} \). The marginal benefit of raising the tax is that public consumption increases, which is valued at \( \frac{\partial g_i}{\partial t_i} \). The equilibrium occurs at the intersection of these two curves. With the above parameters, the Nash equilibrium is \( (t_1^*, t_2^*) = (0.0923, 0.0923) \).

Further, it can be shown numerically (and has been proven theoretically by Bucovetsky (1991) and Wilson (1991)) that if the populations of the two states are different, the smaller state chooses the lower capital tax rate in equilibrium. This is because the elasticity of capital with respect to the tax rate, which is always negative, becomes more negative as that state’s population share falls. The smaller state therefore has more elastic demand for capital and chooses to set its tax rate lower than the large rival state, encouraging a greater rise in its capital-labor ratio.

### 2.2 The Cooperative Equilibrium

Suppose instead that a federal government could set each state’s capital tax rate. The federal government’s maximization problem, then, is to choose the pair of tax rates that maximize the sum of the population-weighted state utilities. Note that, since each state’s consumers’ utility enters its objective, the federal government will account for the externality associated with tax competition; that is, the resulting tax rates, \( (t_1^C, t_2^C) \), are socially efficient.

\(^8\)Note, too, that separability of pollution or environmental equality in utility is not uncommon in the environmental economics literature either; see, for example, Fullerton and Heutel (2007).
The federal government solves

$$\max_{t_1, t_2} s_1 [v(x_1) + g_1] + s_2 [v(x_2) + g_2]$$

subject to (1)–(4) holding for each state $i$. As before, we can simplify the problem by first solving for each state’s allocation as a function of the pair of tax rates, $(t_1, t_2)$. To do this, again combine (1) and (2) to determine $k_1(t_1, t_2)$ and $k_2(t_1, t_2)$. Then, plugging these values into the consumer and the government budget constraints, we can solve for $x_i$ and $g_i$ as functions entirely of the tax rates.

With this simplification, the socially efficient pair of taxes solves the following two first order
conditions:

\[ t_1 : \quad s_1 v_x(x_1) \frac{\partial x_1}{\partial t_1} + s_1 \frac{\partial g_1}{\partial t_1} + s_2 v_x(x_2) \frac{\partial x_2}{\partial t_1} + s_2 \frac{\partial g_2}{\partial t_1} = 0 \]
\[ t_2 : \quad s_1 v_x(x_1) \frac{\partial x_1}{\partial t_2} + s_1 \frac{\partial g_1}{\partial t_2} + s_2 v_x(x_2) \frac{\partial x_2}{\partial t_2} + s_2 \frac{\partial g_2}{\partial t_2} = 0 \]

Notice that these first-order conditions include terms that measure the impact of changing state \( i \)'s tax rate on state \( j \).

**Proposition 1.** *In the socially efficient equilibrium, both states set the same capital tax rate (tax harmonization) and this common tax rate is higher than the optimal capital tax rates derived in the competitive equilibrium.*

Detailed discussions of the two points in this proposition can be found in Bucovetsky (1991) and Wilson (1991). The proposition first states that the socially efficient equilibrium involves tax harmonization. Because the states have identical technologies, tax harmonization implies that each state has the same per-capita capital stocks: \( k_1 = k_2 = k^* \). Consequently, the marginal products of capital are equal across states. If this were not the case, the federal government should change taxes so that capital moves from the low marginal product state to the high marginal product state.

The second result—that the common tax rate in the socially efficient equilibrium is higher than the competitive tax rates, can be seen by comparing the first-order conditions from the federal government’s problem with those from the individual state governments. When the federal government considers a rise in state \( i \)'s tax rate, it perceives the same marginal cost as did that state in the competitive equilibrium: a rise in the tax rate induces an outflow of capital, which lowers private consumption. Now, however, the federal government perceives the marginal benefit to be higher than in the competitive equilibrium. This is because the rise in state \( i \)'s tax rate leads to an increase in the capital-labor ratio of state \( j \), a benefit not valued by state \( i \) in the competitive equilibrium. This additional value is captured in the term \( s_j/s_i \left[ v_x(x_j) \partial x_j/\partial t_i + \partial g_j/\partial t_i \right] \). The result is that the socially efficient tax rates chosen in the cooperative equilibrium are higher than in the competitive equilibrium.

Using the same parameter values and functional forms as above, Figure 2 displays the coopera-
Figure 2: Cooperative Equilibrium

tive equilibrium. Specifically, we have plotted the marginal cost of varying state 1’s tax rate, as well as both the private and the social marginal benefits of that change, assuming that $t_2 = t_2^C$. As can be seen, the social marginal benefit curve is above the private marginal benefit curve, meaning that the federal government chooses a higher tax rate than either state would while acting competitively. For comparison to the competitive case, the equilibrium tax rates are $(t_1^C, t_2^C) = (\frac{1}{5}, \frac{1}{5})$.

3 Model with Permits

Until now we have simply shown that tax competition leads states to implement inefficiently low capital taxes. Two solutions to this inefficiency have been suggested by the tax competition literature: the states should agree to jointly raise their tax rates to their efficient levels or the federal government should impose state-specific Pigouvian taxes on each state.
In this section, we introduce an alternative solution. We propose the creation of a capital tax permits market, where a permit is defined as the right to reduce a state’s capital tax rate by one percentage point from a benchmark value, taking into account the state’s population share. The permits can be sold on a public exchange, populated by state governments. The total number of permits created, the initial distribution of those permits, and the benchmark tax rates are to be set by the federal government. As we will show below, if the federal government supplies the correct number of permits given the benchmark, the resulting price of the permits will fully reflect the size of the externality associated with capital tax competition and the socially efficient outcome will be realized. We characterize this equilibrium in the first subsection, use some numerical examples to illustrate salient features of the model in the second subsection and, in the third subsection, discuss some practical issues.

3.1 Equilibrium

Let \( \{b_1, b_2\} \) be the benchmark pair of tax rates set by the federal government. This benchmark can be any value greater than the socially optimal tax rate and may or may not differ across states; i.e., the benchmark values can be arbitrary functions of population or be set uniformly. Let \( \{q_0^1, q_0^2\} \) be the initial distribution of permits across the states, where \( Q = \sum_{i=1}^{2} q_0^i \). Denote the price of a permit, determined endogenously, by \( p \). After all transactions are finished, the final distribution of permits across states is \( \{q_1, q_2\} \). Each permit held by state government \( i \) allows that government to reduce its capital tax by one normalized percentage point; that is, one percentage point per percentage point in population share of that state. That is, if state \( i \) wants to reduce its tax rate to \( t_i \) from its benchmark, then it would need to hold \( q_i = \frac{b_i - t_i}{n_i} \) permits. Notice, for the same percentage point drop in the capital tax, large states are required to hold fewer permits than small states. The intuition behind this result is seen in the competitive equilibrium: small states have more incentive to reduce their tax rates, and in order to deter this behavior, small states are forced to hold more permits.\(^9\)

\(^9\)This result is analogous to the case of permits for pollution emissions that do not mix uniformly in the atmosphere (for example, this is true for SO\(_2\) pollution). To account for the fact that changes in the policy of one state do not affect the rival state in the same way, transfer coefficients are needed to link tax reductions with the external damages that
With the introduction of the public exchange, state government $i$’s budget constraint is

$$h_i \equiv t_i k_i + p (q_i^0 - q_i) = g_i + p (q_i^0 - q_i)$$

where the second term on the right accounts for the fact that government revenue is a function of the number of permits bought or sold on the exchange. If $q_i < q_i^0$, then state $i$ has sold permits on the market and uses these additional funds to increase its spending on public goods. This increase in public good provision helps defray the cost of maintaining a higher tax rate on capital.

If $q_i > q_i^0$, then state $i$ must purchase additional permits, which decreases its provision of public goods. This additional cost reflects the externality that decreasing the state’s capital tax has on rival states. Finally, the market-clearing condition for permits is

$$Q = \frac{b_1 - t_1}{s_1} + \frac{b_2 - t_2}{s_2}. \quad (6)$$

This market for permits does not affect the representative consumer directly, and so her utility function and budget constraint remain the same as before. Furthermore, the mobility of capital still ensures that (1) holds. Combining this equilibrium condition with (2), we can derive the distribution of capital across states as a function of the pair of tax rates: $k_i (t_i, t_j)$. Given this, the optimization problem for state $i$ is

$$\max_{t_i} v [x_i (t_i, t_j)] + h_i (t_i, t_j)$$

result. One solution proposed in the environmental literature has been zonal permit systems which, in our case, would allow one-for-one trading of permits among states with similar population and would require states with sufficiently different characteristics to trade at a rate that accounts for a factor different from one; see, for example, Tietenberg (1995) for a discussion of these and other related issues concerning tradeable emissions permits. This would help mitigate, again borrowing from the environmental literature, the so-called “hot-spot” problem related to the fact that certain regions pool pollution more than others and so they bear an excessive share of the welfare burden from pollution. In our capital tax setting, this problem would arise when one state acquires a disproportionately large share of the existing permits. Note the difference however, in our setting the driving feature is incentives: certain states have incentive to reduce tax rates more than others; while in the pollution setting, exogenous factors play a role—environmental conditions lead to some unfortunate regions incurring a relatively large cost of the pollution externality. Furthermore, a distinction between zonal trading of pollution permits and that of our capital tax reduction permits is that the exchange rate in the latter case (and, in particular, in our model) is based on observable features of the states (such as population) whereas with pollution permits such transfer coefficients must be estimated or approximated based off how emissions in one region affect an area of interest.
where
\[ h_i(t_i, t_j) = t_i k_i(t_i, t_j) + p \left( q_i^0 - \frac{b_i - t_i}{s_i} \right) \]
\[ x_i(t_i, t_j) = f[k_i(t_i, t_j)] - f'[k_i(t_i, t_j)] k_i(t_i, t_j) + \{ f'[k_i(t_i, t_j)] - t_i \} k^* \]

The first-order condition from this problem is
\[ v_x(x_i) \frac{\partial x_i}{\partial t_i} + \frac{\partial h_i}{\partial t_i} = 0 \] (7)
where \( \frac{\partial x_i}{\partial t_i} \) is the same as above and
\[ \frac{\partial h_i}{\partial t_i} = k_i + t_i \frac{\partial k_i}{\partial t_i} + \frac{p}{s_i} = \frac{\partial g_i}{\partial t_i} + \frac{p}{s_i}. \]

The Nash equilibrium that results from the permit market satisfies the pair of first-order conditions embedded in (7) as well as the permit market-clearing condition (6). Comparing these to the first-order conditions from the federal government’s optimization problem allows us to establish the following theorem:

**Theorem 1.** Given benchmarks \( \{b_1, b_2\} \), if the two states have identical quasi-linear utility functions and if their production functions are quadratic, the equilibrium tax rates that emerge from the permits market are the same as those in the socially efficient equilibrium, as long as the aggregate level of permits \( Q \) is chosen correctly. As such, the permits market achieves social efficiency.

The proof is provided in the appendix so as not to disrupt the flow of the paper. This theorem lays out our paper’s main result: introducing a permits market for capital tax rate reductions induces competitive states to choose the socially efficient level of taxes so long as the number of permits is chosen correctly. A natural implication of this proposition is that the sum of both states’ welfare is higher under the permits market than it is in the competitive equilibrium.

Figure 3 illustrates the solution. In that figure, we have drawn each state’s demand for permits: the left y-axis indicates state 1’s willingness to pay for each level of permit while the right y-axis
is analogous for state 2. The size of the x-axis represents the total number of permits allocated by the federal government, \( Q \). The intersection of the two curves indicates the equilibrium price for the permits, as it is the only price at which \( q_1 + q_2 = Q \). In the graph, the chosen number of permits implies that \((t_1^P, t_2^P) = (\frac{1}{6}, \frac{1}{6})\), the same tax rates as in the cooperative solution.

We have included one additional figure to help draw a comparison across the three equilibria. In Figure 4, we plot the optimal tax rates from each equilibrium for different values of \( L_1 \), holding \( L_2 = 1000 \) and \( k^* = 1 \). When \( L_1 = 1000 = L_2 \), we have the same solutions as cited in the above three graphs. But, as \( L_1 \) grows, the optimal tax rate in the competitive equilibrium for state 1 (2) grows (shrinks). This illustrates Lemma 2 of Wilson (1991) and Theorem 1 of Bucovetsky (1991).\(^{10}\)

Unlike in the competitive equilibrium, as \( L_1 \) grows, the optimal tax rates in the cooperative and the permits market equilibria remain constant. Regardless of the states’ populations, that is, the optimal tax rates in both the cooperative and the permits market equilibria are such that \( t_1 = t_2 = \bar{t} \), where \( \bar{t} \) is greater than the optimal tax rate in the competitive equilibrium. Note that \( \bar{t} \)

\(^{10}\)It follows, that, by Proposition 1 of Wilson (1991) and Theorem 2 of Bucovetsky (1991) the smaller state (which has the lower tax rate in equilibrium) obtains higher utility than its rival (larger) state in a competitive Nash equilibrium.
Figure 4: The Effects of Asymmetries

is independent of the “size” of the asymmetry between the countries. This is because we have fixed $k^*$ across the examples. Were we to instead fix the aggregate level of capital in the economy, then $k^*$ would decrease as $L_1$ increased, which would lead to the efficient tax rate changing with $L_1$. While this would change the implications suggested by this figure, it would not change the results we seek to highlight: first, that the competitive tax rates are too low relative to the optimal, cooperative tax rate and, second, that the permits market is an effective tool in achieving efficiency.\footnote{Note, too, that the efficient policy is optimal from a national perspective but may not necessarily be welfare improving for each individual state—in particular, for the small state. Of course, lump-sum transfers from the large to small state can always be used to guarantee both states are better off in the cooperative solution.}

We can also state the following:

**Corollary 1.** *If the regulatory agency chooses $Q$ inefficiently, a system of tradeable permits with competitive bidding will lead to a post-trading allocation that maximizes a population-weighted sum of states’ welfare; i.e., no other allocation of the $Q$ permits could generate more national welfare than that which obtains under a permit system in the trading equilibrium.*
This result is analogous to that derived by Montgomery (1972) for a pollution permits market. Specifically, Montgomery showed permit systems will minimize the aggregate cost of achieving a given level of pollution. Intuitively, firms abate pollution so long as the cost of doing so is below the price of a permit; to reduce units of pollution that exceed the cost of a permit, they purchase permits. Because all firms face the same price for a permit (the price that prevails in the market), all firms equate their marginal abatement cost and so the aggregate cost of reducing pollution is minimized. In our capital tax permit setting, all states reduce capital tax rates so long as the marginal benefits exceed the cost of a permit. Given that all states face the same price of a permit in equilibrium, the marginal benefits of reducing taxes another percentage point are equated which maximizes national welfare for a given level of permits.

3.2 Numerical Examples

In Table 1, we consider three examples of the model under the assumption of quasilinear utility of the form

\[ u_i(x, g) = \log x + g, \quad i = 1, 2, \]

and quadratic production functions.\footnote{Specifically, in all examples we parameterize the production function by setting \( a = 3/2 \) and \( b = 1/3 \) as we did in the figures used to depict features of the equilibria considered earlier. For capital to be productive, this parameterization requires \( k^* \in (0, 9/4) \). In the scenarios involving permits, we set the benchmark taxes rates in both states to be twice the cooperative tax rate and choose the number of aggregate permits to be the efficient level.} In Example 1 we consider a symmetric model and, in Example 2 and Example 3, we consider two asymmetric models. In all examples, we chose the aggregate (national) population and capital stock so that \( k^* = 1 \). This choice does not affect the spirit of our results and we discuss alternative choices of \( k^* \) later. For each example, we vary the initial permit distribution to see the effects this has on the permits equilibrium. We also compare the utility from the permits equilibrium to the utility levels that obtain from a competitive equilibrium.

The first thing to note from the table is that regardless of the population distribution (which changes across examples) or the initial permit distribution (which changes within each example), the permits equilibrium involves the same capital allocation and taxes rates. The driving feature
<table>
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<tr>
<th>Instance</th>
<th>Initial Permit Distribution</th>
<th>Initial Permit Capital Allocation</th>
<th>Initial Permit Tax Rates</th>
<th>Initial Permit Utility Levels</th>
<th>Competitive Utility Levels</th>
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of the model is that states continue adjusting taxes (trading permits) until \( k_i = k^* \) and, hence, the marginal product of capital is equalized across states. In equilibrium, given the utility function specified above, all states choose to produce the private good until \( x = 1 \). This is because marginal utility of the private good exceeds one until \( x = 1 \), after which it is less than one and public good consumption provides higher marginal utility. This is not because we set \( k^* = 1 \) but is simply an artifact of the utility function we’ve chosen. Our choice is attractive since \( \log(1) = 0 \) so \( u_i = g \), which is why we do not present private and public consumption values in the table—the utility of each state corresponds with the amount of the public good provided and consumed in each state.

Changing the distribution of the population or the way permits are initially allocated essentially results in utility transfers across states. If we think of the population distribution as exogenous, then the permit allocation decision allows for a new policy dimension for the federal government. Specifically, with permit sales providing a second channel of state revenue, giving one state excess permits is a novel form of federal funding since the permits have value in the market. The utility a state gets in the permit equilibrium is monotonically increasing in the share of permits it is initially allocated. Such utility transfers do not affect the efficiency of the permits solution, illustrating a general property of Theorem 1: regardless of the initial permit distribution, the permits equilibrium achieves the socially efficient outcome. That is, the sum of \( u_1 \) and \( u_2 \) is maximized (in our examples at \( 1/3 \)) for each example regardless of how permits are allocated initially. The sum of the competitive equilibrium utility levels suggests that magnitude of the inefficiency is quite small for our parameterizations and the disparity between the two shrinks as states become more asymmetric—although inequality in the competitive solution is more severe than in the permits case.

In all examples the final distribution of the permits is inversely related to the population share as long as the states have common benchmarks. This can be seen from equation (6): given taxes are equalized across states, the quantity of permits a state holds in equilibrium is inversely proportional to its population share. This means, in Example 2, that when state 1 receives \( 2/3 \) of the permits and state 2 receives \( 1/3 \) of the permits, that allocation is the equilibrium distribution of permits—no permits get traded after the initial allocation. Comparing states utility levels in both
the permits and competitive equilibria shows that often times one state prefers the competitive equilibrium to the efficient permit equilibrium. Recall that Wilson (1991) as well as Bucovetsky (1991) characterized situations where the small state prefers the competitive Nash equilibrium to the cooperative, efficient equilibrium. By introducing permits, we see situations in which this is true—for example, in the second row of Example 3 the initial allocation of permits is the equilibrium distribution and yet the small state obtains a higher utility in the competitive equilibrium. The opposite is true in row two of Example 2 which again involves an initial allocation of permits which corresponds with the equilibrium allocation. In that instance, which involves states that are closer to being symmetric, both states prefer the permits equilibrium to the competitive one. Lastly, note that the preference of a particular state can go in both directions depending on the initial permit distribution. That is, while the two rows we highlighted in Example 2 and Example 3, respectively, span the interesting cases noted by Wilson as well as Bucovetsky, the other rows from each of these examples illustrate that either the large or small state can prefer permits to no regulation depending on the initial permit distribution. Intuitively, the federal government can compensate a particular state sufficiently well through the initial permit distribution so that it prefers the permits market to the case with no regulation.

Lastly, we should mention that the choice of $k^*$ does not affect the qualitative predictions of the examples we’ve considered. A higher $k^*$ means a lower marginal product of capital and, hence, higher equilibrium tax rates. It also means there is more capital available in the economy relative to the size of the population. Because states still prefer public consumption once one unit of the private good has been produced, the excess capital is allocated entirely to public good provision which increases the utility of the states in each respective instance of Table 1. Lowering $k^*$ has the opposite effects from those described. In fact, if $k^*$ is sufficiently low, it is optimal for each state to subsidize capital and provide negative amounts of the public good. Such instances would correspond to states running a deficit and paying for firms to locate within state boundaries because the marginal value of private good consumption is so high relative to the utility loss from negative public good consumption. Regardless, the competitive market is still inefficient and the

\[13\] Intuitively, given there is no income or consumption sales tax in our model, a negative value of the public good is like a utility tax.
salient qualitative features of Table 1 (equal capital-labor ratios, tax harmonization, efficiency of the permits model regardless of distribution of permits) are preserved given changes in $k^*$. 

### 3.3 Discussion

While incentive-based policies seek to internalize the externalities associated with tax competition, the decentralized nature of the capital tax permits market has numerous advantages relative to a Pigouvian tax placed on tax reductions.\(^{14}\) First, the permits market does not require the federal government to have large amounts of information about each state’s characteristics, as does a Pigouvian tax. This is particularly valuable given that each state would have the incentive to deceive the federal government in its attempts to manipulate the Pigouvian tax it will be charged.\(^{15}\) Rather, for a given benchmark pair of tax rates, the federal government need only supply the correct amount of total permits to the states. Second, given participation is ensured, the permits market is incentive-compatible, as no state wants to deviate and choose a tax rate that is not equal to its socially efficient level.

Note, too, that an additional benefit of the permits market is that the federal government, in setting the benchmark tax rates and the supply of permits, has some influence over the average tax charged on capital. This influence could help mitigate the cost of vertical tax competition as well as horizontal tax competition. Along the same lines, an indirect benefit of the permits mechanism is that the federal government gains some influence over the total taxes charged to firms across both the state and federal levels. In the current system, the federal government only chooses its own capital income tax rate, taking as given the states’ choices. However, with the permits market, the government can influence the states’ decisions by manipulating the cap on the total number of permits in the system.

On first glance, there are at least two potential disadvantages of this permits solution. The

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\(^{14}\)These forms of incentive-based regulation typically dominate so called command-and-control approaches such as uniform standards (a floor or ceiling imposed on the capital tax of any state), at least in terms of efficiency and, in particular, when asymmetries exist.

\(^{15}\)The characterization of equilibria under permits (quantities) and taxes (prices) under uncertainty presented in the classic work by Weitzman (1974) provides insight into the optimal regulatory instrument to use in such settings. Interestingly, Stavins (1998) noted that “when market-based instruments have been adopted in the United States, they have virtually always taken the form of tradable permits rather than emission taxes.”
first concerns participation—states that maintain relatively low capital income taxes (the smaller states) might refuse to be a part of the system, knowing that they would likely end up paying for the permits. This is, in particular, a problem because if states are sufficiently different in population, the smaller state obtains higher welfare from the competitive equilibrium than from the cooperative equilibrium. The federal government, though, could induce recalcitrant states to join by making a portion of its inter-governmental transfers contingent upon entry. This restriction, if imposed on enough funds, would make it in all states’ best interests to join the permits market. An equivalent method of inducing small states to participate in the permits market would be to grant them more initial permits. If given enough initial permits, the small states would be better off joining than not.

The second potential disadvantage is that businesses might worry that the permits market causes increased uncertainty about tax rates. However, this issue could be addressed by increasing the length of the permit tenure. That is, permits could be redefined as the right to reduce a state’s corporate income tax rate by 1% for five or even ten years. This increased tenure would have implications for the price of the permit and may allow for improved forecastability of tax rates.

An additional worry is that the federal government might not set $Q$, the aggregate quantity of permits, correctly. If this is the case, the resulting equilibrium tax rates are no longer socially efficient. While this is true, the tax rates still exhibit some attractive properties. In particular, because

$$\left[ v_x (x_i) \frac{\partial x_i}{\partial t_i} + k_i + t_i \frac{\partial k_i}{\partial t_i} \right] s_i = -p$$

holds in equilibrium for any value of $Q$, a population-weighted version of the equimarginal principle applies. This implies that, in equilibrium, each state will equate the population-weighted marginal benefit of reducing its tax rate by one more percentage point with the price of a permit. Since all states face the same price, the marginal benefit of further reduction is equalized across states. This implies that the net benefits to society from allowing a certain number of aggregate capital tax reductions (as reflected by the number of permits) are maximized in a permits equilibrium—there is no other allocation of capital tax reductions that provides society with as much benefit.
4 Conclusion

Competition for businesses to locate within state borders creates inefficiencies, if left unregulated, as the equilibrium that obtains involves all states setting capital income tax rates that are too low. While much research has focused on introducing Pigouvian taxes to solve this problem, we suggest an alternative solution: the creation of a permits market for capital income tax reductions. The permit market establishes a cost (or opportunity cost) associated with lowering the tax rate which represents the externality states impose on rivals when making their policy choice. Further investigation allowed us to develop some advantages of our approach. Specifically, it equates the marginal benefit of tax reduction across states, allows for flexibility as states can adjust their tax rates due to statewide economic fluctuations by buying and selling permits, does not require large amounts of information on the part of the federal government, and the policy target objectives (if not achieved initially) can be realized by simply increasing or decreasing the aggregate level of permits.

It would be interesting to consider further the various ways in which permits are initially allocated. While we’ve argued that efficiency can be obtained regardless of this distribution so long as the correct number of total permits is provided, the initial allocation will introduce wealth transfers across states as the establishment of a market attributes value to holding a permit. The canonical auction versus grandfather arguments can be made concerning the initial allocation. In other work (Ghosh, Hubbard, and Svec (2012)) we show how the government can use a sealed-bid auction mechanism in which the efficient level of permits is unknown to the federal government and determined endogenously, but efficiency is guaranteed if a fraction of the revenues are rebated to states as suggested by Montero (2008).

In our model, the national capital-labor ratio was fixed at the national level. The spirit of our investigation has been to determine how states alter the distribution of capital through taxes, given this fixed total supply. We feel the permits solution we advocate has some distinct advantages over the mechanisms that have been proposed. However, it would be interesting to investigate the effects these mechanisms have on attracting new capital (perhaps in a model with endogenous research
and development, technological change, or growth or in a model with foreign direct investment). While beyond the scope of our paper, we feel that further investigation of these issues could be possible in a dynamic or international model and constitutes an important area for future research.

Note, too, that we followed Bucovetsky (1991) by introducing an asymmetry in the model by considering states with different populations. However, it would be instructive to consider the effects of heterogeneity along other dimensions. For example, one can imagine productivity differences across states which might obtain due to natural differences like resources or because of market forces like agglomeration effects.

References Cited


A Appendix

Theorem 1 claims that there exists an equilibrium involving tax harmonization which is a solution to both the social planner’s problem and the model with permits.
Proof. The first-order conditions of the cooperative equilibrium (social planner’s problem) can be expressed as

\[ t_1 : \; s_1 v_x (x_1) \frac{\partial x_1}{\partial t_1} + s_1 \frac{\partial g_1}{\partial t_1} + s_2 v_x (x_2) \frac{\partial x_2}{\partial t_1} + s_2 \frac{\partial g_2}{\partial t_1} = 0 \]

\[ t_2 : \; s_1 v_x (x_1) \frac{\partial x_1}{\partial t_2} + s_1 \frac{\partial g_1}{\partial t_2} + s_2 v_x (x_2) \frac{\partial x_2}{\partial t_2} + s_2 \frac{\partial g_2}{\partial t_2} = 0 \]

Now, given our focus on an equilibrium which equalizes tax rates across states and, given the after-tax rate of return on capital is equal across states since capital is perfectly mobile, the solution involves capital-labor ratios being equalized. This can be seen from equation (1) and obtains because states have access to the same production functions. As such, in this equilibrium \( k_1 = k_2 = k^* = k \).

In particular, in our example involving quasilinear utility and a quadratic production function, \( t = \frac{-bk^2 + ak - 1}{k} \).

Consider now the equilibrium in the permits model which is characterized by the following first-order conditions:

\[ v_x (x_1) \frac{\partial x_1}{\partial t_1} + \frac{\partial g_1}{\partial t_1} + \frac{p}{s_1} = 0 \]

\[ v_x (x_2) \frac{\partial x_2}{\partial t_2} + \frac{\partial g_2}{\partial t_2} + \frac{p}{s_2} = 0 \]

as well as the permits market-clearing condition

\[ \frac{b_1 - t_1}{s_1} + \frac{b_2 - t_2}{s_2} = Q \]

Now using the solution from the cooperative solution that \( t_1 = t_2 = t = \frac{-bk^2 + ak - 1}{k} \), it remains to be shown that there exists a \( Q \) such that this value of \( t \) is an equilibrium.

Under our parameterization, the above first-order conditions in a competitive model with per-
mits become

\[
\frac{1}{x_1} \left\{ s_2 [k^* - k_1] - k^* \right\} + k_1 - \frac{t_1 s_2}{2b} + \frac{p}{s_1} = 0
\]

\[
\frac{1}{x_2} \left\{ s_1 [k^* - k_2] - k^* \right\} + k_2 - \frac{t_2 s_1}{2b} + \frac{p}{s_2} = 0
\]

\[
\frac{b_1 - t_1}{s_1} + \frac{b_2 - t_2}{s_2} = Q.
\]

Given tax rates are equal, the first two equations are redundant in that they both imply the same value of \( p \) while the last equation can be solved for \( Q \). Specifically,

\[
p = \left( \frac{-bk^2 + ak - 1}{k} \right) \frac{s_1 s_2}{2b}
\]

while

\[
Q = \frac{s_2 b_1 + s_1 b_2 - \left( \frac{-bk^2 + ak - 1}{k} \right)}{s_1 s_2}
\]

Thus, the solution that solves the cooperative equilibrium is the same solution that solves the permits market for an appropriately-set level of permits.

\[\square\]