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Abstract

We present a solution to the Backus-Smith puzzle that, instead of relying on extreme parameter values or complex modeling assumptions, simply switches the framework from infinitely lived agents to overlapping generations. Young agents face non-diversifiable wage risk that leads to a low degree of risk sharing within each country. Subsequently, international price movements are not sufficient to achieve the high consumption-real exchange rate correlation produced in standard infinitely lived agent DSGE models.

JEL Classification Codes: D52, F21, F41, G11

Keywords: Backus-Smith, international portfolio choice, overlapping generations, risk sharing

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1 Introduction

A low (or even negative) consumption-real exchange rate (RER) correlation in international data has been difficult to reproduce in standard two-country dynamic stochastic general equilibrium (DSGE) models driven primarily by technology shocks, a challenge referred to as the Backus-Smith puzzle (Backus and Smith, 1993). The high degree of international consumption risk sharing in DSGE models is broadly independent of the financial market structure and results primarily from international price movements that transmit country-specific gains across borders (Cole and Obstfeld, 1991). Among the multiple approaches aimed to reconcile the theory with the data, Corsetti, Dedola, and Leduc (2008) and Enders and Müller (2009) combine incomplete financial markets with low import-export elasticity of substitution; Benigno and Thoenissen (2008) incorporate nontraded goods into their model; and Mykhaylova and Staveley-O'Carroll (2014) add high levels of international debt denominated in foreign currency.

We propose a different approach to bring theoretical outcomes closer to the data by switching the model structure from infinitely lived agents (ILA) to overlapping generations (OLG). In the latter, each country is populated by two cohorts of agents: the young cohort earns an exogenous wage, invests in a variety of assets, and consumes a basket of home and foreign goods, while the old cohort simply consumes the stochastic proceeds of its investment portfolio. The wage risk faced by the young workers cannot be shared with the retirees, significantly reducing within-country risk-sharing. The resulting correlation between the relative aggregate national consumptions and the RER is much closer to (and for some calibrations in range of) empirically calculated values. Our results hold despite the presence of a fairly comprehensive array of financial assets and are robust to changes in several key parameter values.

2 Model

The OLG framework is a simplified version of that in Staveley-O'Carroll and Staveley-O'Carroll (2017). Below we outline only its most salient features and direct the interested reader to the original paper for details of the solution and calibration methodology.

In the home economy, dividends $d_{h,t}$ and wages $w_{h,t}$, both measured in terms of the home good, evolve according to independent exogenous two-state Markov shock processes; the foreign economy is modeled symmetrically, its variables denoted with the subscript f. A cohort born in period t maximizes its lifetime utility

$$v_t^{h,t} \equiv \frac{\left(c_t^{h,t}\right)^{1-\gamma}}{1-\gamma} + \beta E\left[\frac{\left(c_{t+1}^{h,t}\right)^{1-\gamma}}{1-\gamma}\right] \tag{1}$$

where c is composed of home and foreign goods,[1] β is the intertemporal discount factor, γ is the coefficient of relative risk aversion (CRRA), and superscripts t and h indicate the period and place of birth, respectively. The maximization is subject to two intertemporal

[1] We use a standard CES aggregator $c_t^h = \left[\phi_{h,t}^{1-\sigma}\left(c_{h,t}^h\right)^{\sigma} + \left(1-\phi_{h,t}\right)^{1-\sigma}\left(c_{f,t}^h\right)^{\sigma}\right]^{\frac{1}{\sigma}}$ with a stochastic home bias process $\phi_{h,t}$ and a constant elasticity of import-export substitution $\frac{1}{1-\sigma}$.

budget constraints:

$$c_t^{h,t} + b_{h,t}^{h,t} + q_t b_{f,t}^{h,t} + q_{h,t} a_{h,t}^{h,t} + q_t q_{f,t} a_{f,t}^{h,t} \leq p_{h,t} w_{h,t} \quad (2)$$

$$c_{t+1}^{h,t} \leq \Omega_{t+1}^{h,t} \quad (3)$$

where $b_{h,t}^{h,t}$, $b_{f,t}^{h,t}$, $a_{h,t}^{h,t}$, and $a_{f,t}^{h,t}$ are the young cohort's holdings of the home and foreign bonds and equities, respectively, q_t is the RER, $q_{h,t}$ and $q_{f,t}$ are the prices of the home and foreign equities, respectively, and $p_{h,t}$ is the price of the home good. Each country has one unit of stock, and both bonds are in zero net supply. The home agent's second-period wealth $\Omega_{t+1}^{h,t}$ is composed of the returns on her portfolio investments:

$$\begin{aligned} \Omega_{t+1}^{h,t} \equiv & \left(1 + r_{h,t}\right) b_{h,t}^{h,t} + q_{t+1} \left(1 + r_{f,t}\right) b_{f,t}^{h,t} + \\ & \left(q_{h,t+1} + p_{h,t+1} d_{h,t+1}\right) a_{h,t}^{h,t} + q_{t+1} \left(q_{f,t+1} + p_{f,t+1} d_{f,t+1}\right) a_{f,t}^{h,t} \end{aligned} \quad (4)$$

where $r_{h,t}$ and $r_{f,t}$ are the real yields on the home and foreign bonds, respectively.

2.1 Infinitely-lived agents

We compare the OLG setup to the fairly standard two-country ILA DSGE model with incomplete financial markets by modifying the above framework as follows. The home agent maximizes her lifetime utility

$$\nu_t^h \equiv \sum_{i=0}^{\infty} \beta^i E \left[\frac{\left(c_{t+i}^h\right)^{1-\gamma}}{1-\gamma} \right] \quad (5)$$

subject to

$$c_t^h + b_{h,t}^h + q_t b_{f,t}^h + q_{h,t} a_{h,t}^h + q_t q_{f,t} a_{f,t}^h \leq \Omega_t^h \quad (6)$$

where

$$\begin{aligned} \Omega_t^h \equiv & \; p_{h,t} w_{h,t} + \left(1 + r_{h,t-1}\right) b_{h,t-1}^h + q_t \left(1 + r_{f,t-1}\right) b_{f,t-1}^h + \\ & \left(q_{h,t} + p_{h,t} d_{h,t}\right) a_{h,t-1}^h + q_t \left(q_{f,t} + p_{f,t} d_{f,t}\right) a_{f,t-1}^h \end{aligned} \quad (7)$$

The main distinction between the ILA and the OLG models is that in the former the agents receive both wage and dividend income in each period, whereas in the latter households receive wages while young and dividends when old.

2.2 Calibration and solution

Both countries are calibrated symmetrically, with each period representing 25 years. We set $\beta = 0.375$ to achieve the 4% annual real rate of return. We assume that home and foreign goods are imperfect substitutes in consumption, $\sigma = 1/3$, and let the home bias parameters evolve as $\phi_{i,t} = 0.75 + e_t$, with $e_t = \{-0.01, 0.01\}$ and $i = \{h, f\}$.[2] We set $\gamma = 1$, corresponding to log utility of consumption. We use the moments of the annual series for U.S. compensation of employees and capital income to calibrate the Markov processes

[2] Having more shocks (five in our model) than assets (four) is a sufficient condition for asset market incompleteness. We calibrate the home bias shock to be relatively small to ensure that it is not the main driver of our results.

	ILA	OLG
Panel A: risk sharing		
$Corr\left(\frac{c_t^h}{c_t^f}, q_t\right)$	0.81 (0.129)	0.37 (0.027)
$Corr\left(c_t^{i,t}, c_t^{i,t-1}\right)$ [a]		0.36 (0.026)
Panel B: portfolio investment		
$a_{h,t}^{h,t}$	0.99 (0.007)	0.50 (0.003)
$a_{f,t}^{h,t}$	0.01 (0.007)	0.50 (0.003)
$b_{h,t}^{h,t}$	0.19 (0.003)	0.00 (0.000)
$b_{f,t}^{h,t}$	−0.19 (0.002)	0.00 (0.000)

Table 1: We report averages across 1000 simulations, with standard errors shown in parentheses.
Notes: a) Within-country risk sharing is reported for $i = \{h, f\}$.

for wages and dividends. For simplicity, we set all (within and across country) wage and dividend correlations to zero.

The model is solved globally over a compact state-space grid. Given the structural complexity of the portfolio choice problem, we rely on numerical simulations to study the extent of risk sharing. To obtain the results below, we simulate each model 1000 times starting at the symmetric time-zero cross-country wealth distribution. Each simulation is 1100 periods long, with the first 100 periods discarded so that the initial conditions do not impact the findings.

3 Results

The focus of our study is the correlation between the RER q_t and the relative home and foreign (aggregate) consumptions c_t^h and c_t^f, defined as $c_t^i = c_t^{i,t} + c_t^{i,t-1}$ for $i = \{h, f\}$ in the OLG framework. In models with perfect international risk sharing this correlation is equal to unity, much higher than in the international data and thereby giving rise to the Backus-Smith puzzle.[3] To better understand the puzzle and the possible ways to address it, consider that a positive (home) output shock produces both a supply and a demand effect as more (home) goods are available for purchase and (home) agents are simultaneously made wealthier. In standard ILA DSGE models, the supply effect (which lowers the international price of the home good) usually outweighs the demand effect (which instead drives up the price of the home good). The resulting RER depreciation transfers some of the home wealth gain abroad, improving risk sharing even if international financial markets are incomplete.

Panel A of Table 1 shows our main finding: *ceteris paribus*, the ILA model produces more international risk sharing than the OLG framework. The key mechanism driving this OLG outcome is the lack of within-country risk-sharing, equal to 0.36 in both economies.

To help understand these results, panel B of Table 1 lists the optimal portfolios in each framework. The representative ILA consumer faces nontradable wage risk and two stochastic dividend streams denominated in different currencies; all shocks are uncorrelated.

[3] We calculated the relative consumption-RER correlations vis-à-vis the U.S. for 132 countries during the 1970–2016 period. The one-standard-deviation range around the mean of this correlation is $[-0.67; 0.32]$. Details and the description of the data are available from the authors upon request.

Following a high realization of the home wage (i.e., more home output), the RER depreciates, benefitting the foreign agent. The home consumer adjusts her bond holdings (long in home, short in foreign) to amplify this mechanism: the RER depreciation increases the cost of repaying foreign debt, further improving the extent of (wage income) risk sharing.[4] Concurrently, she holds almost the entirety of the home equity. This portfolio selection effectively reduces the number of endowment shocks facing her from three ($d_{h,t}$, $d_{f,t}$, and $w_{h,t}$) to one: relatively high or low home output (via any combination of home wage and dividend outcomes). The setup becomes isomorphic to the standard two-country DSGE models driven by productivity shocks, in which having a flexible RER is sufficient to achieve a high degree of risk sharing.[5]

The OLG model, on the other hand, produces the well-known two-country Lucas tree outcome (Lucas, 1982). Since the young agents are not exposed to the wage risk in the second period of their lives, they invest in equal portions of home and foreign equity to perfectly share the second-period dividend income. It is crucial to note that the two generations within a country cannot share wage risk: the young cohort is born into a particular realization of wage income, at which point it is too late to diversify it via financial markets. Thus, bonds are superfluous for the purposes of risk sharing. Outside of financial markets, there is only one relative price left—the RER—to transfer risk among the four heterogeneous groups of agents. Unsurprisingly, the resulting extent of risk sharing is quite low both within and across national borders.

3.1 Robustness checks

Trade elasticity $\frac{1}{1-\sigma}$ can significantly affect the extent of cross-border wealth transfer via the RER. The parameter estimates in the literature, however, range from $\sigma = -3.55$ (complements) to 0.83 (substitutes).[6] As home and foreign goods are made more complementary (setting $\sigma = -1/3$), RER volatility increases: an ever larger RER fluctuation is necessary to shift demand between the two goods. A more volatile RER, in turn, is more effective at transferring wealth internationally and thus increases risk sharing in the ILA framework. Consumers no longer need to bolster the RER wealth transfer effect by shorting foreign bonds. In fact, bond holdings are reversed relative to the baseline: home consumers borrow in local currency to invest in foreign bonds ($b_{h,t}^{h,t} = -0.10$ and $b_{f,t}^{h,t} = 0.10$).[7] Young agents in the OLG model, on the other hand, hold no assets when they are hit with the wage shock; thus, there is no relative financial wealth to transfer via the RER. Since consumers do not shift to the cheaper good as readily as in the baseline, the overall risk sharing falls.

In the second exercise, we increase the CRRA coefficient to $\gamma = 3$.[8] This modification strengthens the demand effect of the endowment shocks (for example, an increase in the home wage): a greater desire to smooth consumption intertemporally drives up demand for financial instruments and consequently raises their price. In turn, rising asset prices increase

[4] Mykhaylova and Staveley-O'Carroll (2014) show that nonzero holdings of foreign currency denominated debt can significantly alter the extent of international risk sharing.

[5] The baseline ILA calibration produces a somewhat lower correlation than the standard DSGE models since we do not model capital accumulation. This simplifying assumption weakens the supply effect, since current output gains do not translate to more production in future periods.

[6] Bodenstein (2010).

[7] Aside from this change in the portfolio composition of the ILA agents, the robustness exercises result in virtually the same equity and bond holdings as reported in Table 1; we therefore omit them for space considerations.

[8] For example, Eichenbaum et al. (1988) report a range of 0.5–3.

$Corr\left(\frac{c_t^h}{c_t^f}, q_t\right)$	**ILA**	**OLG**
Baseline [a]	0.81 (0.129)	0.37 (0.027)
$\sigma = -1/3$	0.91 (0.078)	0.21 (0.035)
$\gamma = 3$	0.53 (0.095)	0.17 (0.027)
$Corr\left(d_{i,t}, w_{i,t}\right) = 0.457$ [b]	0.84 (0.118)	0.64 (0.021)
$Corr\left(d_{h,t}, d_{f,t}\right) = 0.7$	0.77 (0.144)	0.53 (0.020)

Table 2: We report averages across the 1000 simulations, with standard errors shown in parentheses.
Notes: a) Taken from panel A of Table 1.
b) We modify the correlation symmetrically for both economies, $i = \{h, f\}$.

the wealth of the (home) asset holders and thereby strengthen their demand (for the more abundant home good). Consequently, the RER response is muted and the resulting extent of risk sharing lowered in both ILA and OLG specifications. Nonetheless, our basic conclusions remain unchanged: OLG framework delivers a significantly lower level of risk sharing.

We set the within-country wage-dividend correlation to 0.457 to match the corresponding U.S. empirical moment. This change improves risk sharing in both frameworks (marginally in ILA, significantly in OLG) and makes it more similar in the two models. The explanation is quite straightforward: as mentioned in the introduction, flexible RER is generally sufficient to achieve a high degree of international risk sharing in the presence of country-specific productivity shocks. By increasing the wage-dividend correlation in the OLG framework, we work to "merge" the two endowment shocks into one production shock (something the ILA consumers can do via portfolio choices), enabling the RER to better transmit wealth across borders.

Finally, we increase the cross-country dividend correlation to 0.7.[9] Risk sharing decreases marginally in the ILA framework; in the OLG model, on the other hand, risk sharing improves vis-a-vis the baseline—but still remains below the ILA level. Recall that in the ILA model, RER is generally very effective at transferring risk between countries, and so the extend of cross-country dividend correlation is virtually irrelevant for this purpose. In the OLG model, the RER ability to transfer risk is inhibited, and so a closer alignment of the two shocks processes delivers a positive effect on consumption risk-sharing.

4 Conclusions

We propose a way to address the Backus-Smith puzzle without extreme parameterizations or complex model specifications by adding agent heterogeneity and imperfect risk sharing within each country. When consumers cannot effectively pool risks locally, the RER fails as an efficient mechanism for international risk sharing. This result emerges despite the presence of a wide array of international financial assets. We achieve this outcome by modeling an OLG setting with nondiversifiable (young cohort) wage risk. However, our findings can likely be generalized to other settings where agents cannot share risks perfectly within a country, such as including a myopic consumers or imposing credit constraints on a fraction of agents.

[9] Calibration choice taken from Staveley-O'Carroll and Staveley-O'Carroll (2017).

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