

8-1-2013

Endogenous Voting Weights for Elected Representatives and Redistricting

Justin Svec

College of the Holy Cross, jsvec@holycross.edu

James Hamilton

College of the Holy Cross, jbhami15@g.holycross.edu

Follow this and additional works at: http://crossworks.holycross.edu/econ_working_papers

 Part of the [Economics Commons](#)

Recommended Citation

Svec, Justin and Hamilton, James, "Endogenous Voting Weights for Elected Representatives and Redistricting" (2013). *Economics Department Working Papers*. Paper 145.

http://crossworks.holycross.edu/econ_working_papers/145

This Working Paper is brought to you for free and open access by the Economics Department at CrossWorks. It has been accepted for inclusion in Economics Department Working Papers by an authorized administrator of CrossWorks.

Endogenous Voting Weights for Elected Representatives and Redistricting

By

Justin Svec and James Hamilton

July 2013

COLLEGE OF THE HOLY CROSS, DEPARTMENT OF ECONOMICS
FACULTY RESEARCH SERIES, PAPER NO. 13-07*



Department of Economics
College of the Holy Cross
Box 45A
Worcester, Massachusetts 01610
(508) 793-3362 (phone)
(508) 793-3708 (fax)

<http://www.holycross.edu/departments/economics/website>

* All papers in the Holy Cross Working Paper Series should be considered draft versions subject to future revision. Comments and suggestions are welcome.

Endogenous Voting Weights for Elected Representatives and Redistricting

By

Justin Svec¹
College of the Holy Cross

and

James Hamilton
College of the Holy Cross

July 2013

Abstract

This paper analyzes the merits of a novel method of eliminating the power of a gerrymanderer that involves an endogenous weighting system for elected representatives. This endogenous weighting system ties the voting weight of elected representatives in the legislature to the share of the voters who voted for that representative's party and to the share of representatives elected from that party. If the weights are set correctly, it can be shown in simple voting models like Gilligan and Matsusaka (1999) that redistricting has no influence on the policy passed by the legislature. This benefit, though, is out-weighed by the fact that, in more realistic voting models, the gerrymanderer can manipulate the redistricting process to achieve greater policy bias than under the status quo.

JEL Classification Codes: D72, H11

Keywords: Redistricting, representative weights, gerrymander

¹ Department of Economics, Box 87A, College of the Holy Cross, Worcester, MA 01610-2395, 508-793-3875 (phone), 508-793-3708 (fax), jsvec@holycross.edu

1 Introduction

Once a decade, after the results from the US Census are released, state legislatures update district lines to reflect changes in the population. These legislatures face two constraints when modifying district lines. The first constraint is that each district must have roughly the same number of people within its borders, and the second constraint is that the districts must be contiguous. In addition to these legal constraints, there are informational and geographical limitations as well. These limitations further reduce the scope of a legislature to gerrymander.

Even given these constraints, the state legislatures still have considerable flexibility in determining the position of these lines. This flexibility has given legislators the opportunity to redraw district lines for personal gain, often helping to ensure that either incumbents or representatives from their party are elected in future elections.

There has been considerable research devoted to understanding the consequences of gerrymandering. Theoretical analyses including Gilligan and Matsusaka (2006), Gul and Pesendorfer (2010), King (1989), and Shotts (2002) show that optimal redistricting leads to partisan bias. This partisan bias implies that the translation between the vote share received by a political party and the number of seats obtained by that party depends on the party in question. Further, this partisan bias has been shown to lead to policy bias (Gilligan and Matsusaka (1999)), a situation in which the resulting policy differs from the preferred policy choice of the median voter. Several empirical studies have supported these conclusions, including Gelman and King (1990), Gilligan and Matsusaka (1999), King and Browning (1987), and Kousser (1996). These papers, focusing on US data, find both substantial policy bias in the states and that gerrymandering has sometimes worsened that bias.

Given the potential costs of gerrymandering, a number of researchers have proposed possible solutions. These solutions typically remove the gerrymanderer completely or they reduce the flexibility with which the gerrymanderer can redraw district lines. One solution, implemented in 2008 by California, is to have a neutral committee draw the district lines. While this solution seems appealing, each member of the committee is likely to have a bias in favor of a particular party, calling into question whether the committee as a whole can act with neutrality.¹ In response, some have called for computers to randomly draw district lines. However, as Gilligan and Matsusaka (2006) show, random redistricting does not eliminate partisan bias, except under an extreme and unrealistic set of assumptions.

Gilligan and Matsusaka (2006), in the same paper, also propose adding an additional constraint to the process of gerrymandering. If a gerrymanderer, they suggest, is forced to maximize the homogeneity of voters within a district, then it can be shown that the gerrymanderer's choice results in no partisan

¹In fact, King (1989) and Kousser (1996) provide evidence that within certain states, partisan bias increased after a neutral committee redrew the district lines.

bias. One final solution discussed in the literature is to create districts that elect multiple representatives, although there is a concern that these at-large districts reduce minority representation.

In this paper, we explore the merits of an alternative solution. This solution does not limit the ability of a gerrymanderer to redraw district lines, but instead it reduces the benefits that come from gerrymandering. It accomplishes this by endogenizing the voting weight that each elected representative is accorded when in the legislature. Specifically, the solution sets the weight of each representative elected equal to the vote share received by the representative’s party in the entire state divided by the share of representatives elected from that same party. This endogenous weighting system ensures that a party’s total weight in the legislature is independent of the number of representatives elected from that party.

If one introduces this endogenous weighting system into the simple voting model of Gilligan and Matsusaka (1999), it is straight-forward to show that the gerrymanderer has no influence on the state’s policy outcomes. That is, the endogenous weighting system guarantees that there is no policy bias, regardless of the size of the partisan bias. Moreover, the resulting voting weight of each party is proportionally responsive, meaning that for each 1% increase in a party’s vote share in the state, the party’s total weight in the legislature rises by 1%.

In section 2 of this paper, we define our weighting system and explore some of its key properties. Section 3 analyzes the merits of this endogenous weighting system. As we will show, there are severe drawbacks to this weighting system, drawbacks sufficiently large as to undermine the benefits of the solution. Section 4 concludes.

2 Model

The simple voting model described below follows Gilligan and Matsusaka (1999).

Consider a political jurisdiction with N voters, where for simplicity N is assumed to be an odd number. The voters are divided into two types: there are N_D voters of type D and N_R voters of type R , where $N_D + N_R = N$. The voters of type D prefer Democratic candidates, while the voters of type R prefer Republican candidates. All voters within each type are assumed to be homogenous. Let $V_D \equiv \frac{N_D}{N}$ be the vote share obtained by all the Democratic candidates in the jurisdiction and $V_R \equiv \frac{N_R}{N}$ be the vote share obtained by all the Republican candidates in the jurisdiction.

The political jurisdiction is divided into K , single-representative districts. Each district is constrained to have the same number of voters, meaning that the population in each district is $\frac{N}{K}$. We assume that K is such that $\frac{N}{K}$ is an integer. District $i \in \{1, 2, \dots, K\}$ contains $N_{D,i}$ Democratic voters and $N_{R,i}$ Republican voters, where $\sum_{i=1}^K N_{D,i} = N_D$ and $\sum_{i=1}^K N_{R,i} = N_R$. There are two political parties that sponsor a candidate in each district: the Democratic party and the Republican party. The voters

in each district elect one of these representatives to represent them in the legislature in a winner-take-all election. All representatives within each party are assumed to be homogenous. Let D be the total number of Democratic representatives elected across all districts, and R be the total number of Republican representatives elected across all districts.

A gerrymanderer chooses how to allocate voters across the districts. The gerrymanderer understands that district i will elect a Democratic representative if there are $N_{D,i} \geq \frac{N+1}{2}$ Democratic voters within that district and a Republican candidate otherwise.² Given any allocation, we can determine the resulting partisan bias of the choice. Partisan bias is defined as

$$\beta = \ln \left(\frac{\frac{D}{K}}{1 - \frac{D}{K}} \right)$$

when the vote shares of the two parties are roughly equal. This value measures the size of the discrepancy in representatives elected relative to the voters' preferences.

2.1 The efficient gerrymander

Suppose a gerrymanderer's objective is to allocate voters across the districts in order to maximize the number of representatives elected from her own party. As is well-known in the literature, the gerrymanderer will create as many districts where the voters from her own party enjoy the smallest possible majority. In particular, assuming without loss of generality that the gerrymanderer is a Democrat, she will create as many districts as possible with $\frac{N+K}{2K}$ Democratic voters and $\frac{N-K}{2K}$ Republican voters. This allocation is efficient because it creates the smallest possible majority for Democrats in the most districts.

As is well-known in the literature, the efficient gerrymander leads to high levels of partisan and policy bias. Following the results in Gilligan and Matsusaka (1999), we can show that the maximum bias created by the efficient gerrymander is

$$\beta = \ln \left[\frac{N+1}{K-1} \right]$$

This bias is increasing in N and decreasing in K .

2.2 An alternative weighting system

In this section, we introduce and analyze an alternative weighting system for representatives.

Before we get to the definitions of the weights and our main results, it is helpful to understand what we mean by weighting system. Under the current system, each elected representative gets 1 vote when deciding policy. That is, if D Democratic candidates and R Republican candidates are elected to the

²We assume that there are always sufficient voters of the minority party that a gerrymanderer cannot allocate voters across districts so that only one party's representatives are elected.

legislature, then when the legislature votes on a particular policy proposal, each representative receives one vote. This means that the Democratic party, in total, gets D votes, while the Republican party, in total, gets R votes. As a consequence, if $D > R$, then the outcome of the policy debate is resolved in favor of the Democrats. By altering the allocation of the voters across districts, a gerrymanderer can bias policy outcomes in favor of one party by increasing the number of representatives elected from that party. In effect, partisan bias causes policy bias.

Suppose, though, that the weighting system is no longer one in which all representatives receive one vote. Under this type of alternative, particular candidates could receive more than 1 vote, meaning that they would have a relatively large influence on the policy outcome, while other candidates could receive less than 1 vote, meaning that they would have a relatively small influence on the policy outcome. As we will show, our proposed endogenous weighting system leads to no policy bias and to proportional representation in this simple model, regardless of the gerrymanderer's allocation of voters across districts.

Define the endogenous weighting system to be as follows: let the weight of each Democratic candidate elected be w_D and the weight of each Republican candidate elected be w_R , where

$$\begin{aligned} w_D &= \frac{V_D}{\frac{D}{D+R}} \\ w_R &= \frac{V_R}{\frac{R}{D+R}} \end{aligned}$$

Notice that the weight of a particular candidate depends on two factors: the vote share of the candidate's party within the entire political jurisdiction and the representative share of the party. Specifically, the weights are increasing in the first factor and decreasing in the second.

As we will argue, the relationship between the weight and the factors is normatively appealing. First, consider the fact that the weight of the Democratic party, for example, depends positively on the vote share earned by the Democratic party. This dependence is beneficial in that the Democratic party's influence on policy outcomes depends positively on how many voters vote for the Democratic party. Second, consider the fact that the weight of the Democratic party falls as more Democratic representatives are elected. This dependence is beneficial because it undermines the benefits of gerrymandering. That is, if a gerrymanderer chooses a redistricting that increases the representatives elected from her party, this redistricting will also reduce the weight given to each representative elected. As such, the gerrymanderer cannot bias the policy outcomes through the redistricting process, even if she can still achieve partisan bias.

Now, given the weighting system, we can describe our main results in the following propositions:

Proposition 1 $w_D D + w_R R = D + R$

Proof.

$$\begin{aligned}w_D D + w_R R &= \frac{V_D}{D+R} D + \frac{V_R}{D+R} R \\ &= V_D (D+R) + V_R (D+R) \\ &= D + R\end{aligned}$$

■

This proposition states that, regardless of the voter allocation chosen by the gerrymanderer, the weighting system does not change the total number of representative votes in a legislature. That is, if the legislature contains X representatives, then under the weighting system, the total number of votes remains X .

Proposition 2 *The weighting system results in no policy bias.*

Proof. To show that the weighting system results in 0 policy bias, we need to show that $w_D D = V_D (D + R)$.

$$\begin{aligned}w_D D &= \frac{V_D}{D+R} D \\ &= V_D (D + R)\end{aligned}$$

■

This proposition highlights an important point: whereas the efficient gerrymanderer can achieve a substantial policy bias under the current system, the gerrymanderer has no power to alter the total weight of her party in the legislature under the endogenous weighting system. This is because the total weight accorded to a party depends entirely on the vote share received by that party. Moreover, as the above proof makes clear, the gerrymanderer's redistricting choice results in no policy bias *for any value of V_D* .

Proposition 3 *A party's total weight in the legislature is proportionally responsive to the share of the vote received by that party over the relevant range of vote shares.*

Proof. Assume that there are always sufficient voters of the minority party that a gerrymanderer cannot alter district lines so that only one party's representatives are elected.³ The proposition states that for each 1% rise in the vote share of a particular party, that party's weight in the legislature rises by 1%.

³This is the 'relevant range,' as mentioned in the proposition.

Put another way, we must show that the elasticity of the party's weight to vote share is equal to unity.

$$\begin{aligned}
\epsilon_{w_D D, V_D} &= \frac{\partial w_D D}{\partial V_D} \frac{V_D}{w_D D} \\
&= \frac{D}{D+R} \left(\frac{V_D}{\frac{V_D}{D+R} D} \right) \\
&= D + R \left(\frac{1}{D + R} \right) \\
&= 1
\end{aligned}$$

■

This proposition follows from the previous: because a party's weight depends entirely on the vote share received by the party, if the vote share rises or falls, the party's weight rises or falls by the same amount.

Taken together, we have shown that the endogenous weighting system does not alter the total number of votes cast in the legislature, but rather it alters the distribution of the votes in such a way as to eliminate the power of the gerrymanderer to influence the policy outcomes.

3 Discussion

In this section, we discuss the potential merits of this endogenous weighting system.

The most direct benefit of the weighting system is that, at least in this simple model, the gerrymanderer can no longer influence policy outcomes by manipulating the district lines. This is because the total weight of the representatives from each party depends entirely on the vote share obtained by that party and not on the number of representatives elected. As such, the total weight given to each political party is not manipulable. Put simply, the weighting system ensures that gerrymandering is no longer productive.

An additional benefit of the endogenous weighting system is that it would encourage more voters to vote. To see this, compare the current system with the proposed endogenous weighting system. Under both systems, a voter chooses to vote when the benefit of voting outweighs the cost. One factor driving the benefit of voting is the likelihood of the voter being pivotal in the election. Under the current system, the probability of being pivotal varies inversely with the total number of voters. Consequently, it is optimal for each voter to employ a mixed strategy when voting, voting with some probability p and not voting with probability $1 - p$. Under the endogenous weighting system, though, the probability of being pivotal rises. This is because the voter's vote influences the weight given to the party's representatives, even if the voter's preferred representative is not elected. This additional incentive should lead more people to vote.

There are a number of drawbacks associated with this endogenous weighting system. One drawback is that, if this weighting system does in fact eliminate the influence of a gerrymanderer, then it would be

doubtful that the party in power would enact the policy. This implies that this weighting system, as well as any other possible solution to the problem of gerrymandering, is unlikely to be implemented.⁴

A second, and perhaps more severe, drawback is that, if we assumed a more realistic model, the endogenous weighting system can lead to greater policy bias than the current system. To show this, we will construct a simple model that involves greater voter heterogeneity. Suppose that there are 9 voters and 3 districts. The preferred policy points of the voters are distributed across a uni-dimensional policy space. For simplicity, assume that voter i prefers that a policy has value i , meaning that the median voter's preferred policy is 5. The first 5 voters are labeled Democrats, and the last 4 voters are labeled Republican. If the gerrymanderer is a Democrat, the natural question is 'how large a policy bias can the gerrymanderer achieve under the two systems?'

First, let us consider the current system in which all representatives receive one vote. The efficient Democratic gerrymander would involve putting voters 1,2, and 9 in district 1, voters 3,4, and 8 in district 2, and voters 5,6,and 7 in district 3. The representative chosen in each district would then reflect the preferred policy of each district's median voter, meaning that the representative in district 1 would have a preferred policy of 2, the representative in district 2 would prefer 4, and the representative in district 3 would prefer 6. The median representative, then, would have a preferred policy of 4, implying that the policy enacted would be 4. The resulting policy bias is therefore 1.

Now, consider the endogenous weighting system. The efficient Democratic gerrymander in this case would involve putting voters 1,2, and 3 in district 1, voters 4,6, and 7 in district 2, and voters 5,8, and 9 in district 3. The representatives in the three districts would then prefer 2,6, and 8. However, the representatives would not have equal weight when deciding policies in the legislature. In fact, the Democratic representative would have a weight of $\frac{5}{3} \left(= \frac{5}{\frac{2}{3}} \right)$, while each Republican representative would have a weight of $\frac{2}{3} \left(= \frac{4}{\frac{3}{2}} \right)$. Since the one Democratic representative has more than half the vote in the legislature, that representative can determine the policy outcome by herself. Thus, the resulting policy bias is 3, a larger bias than under the current system with fixed weights.

To understand why the gerrymanderer was able to achieve a greater policy bias under the endogenous weights, note that the weights are designed to undermine the power of the gerrymanderer by increasing the voting weight of a party's representatives hurt by the redistricting and decreasing the voting weight of a party's representatives helped by the redistricting. Knowing this, though, the gerrymanderer can take advantage of the algorithm by purposely reducing the number of representatives elected by her own party. This is an effective way to increase the weight of a few representatives. At the extreme, the gerrymanderer

⁴Of course, we do see examples of states that try to reduce the power of the gerrymanderer. In fact, numerous states have recently moved to a system in which a neutral committee decides how to draw the district lines, including Washington, California, and Arizona.

can allocate voters so that only one representative is elected from her party, giving the entire voting weight of the party to that one legislator. Further, if the voting share of the gerrymanderer's party is greater than 50%, then that one representative becomes a dictator, deciding all policies in her favor.

4 Conclusion

In this paper, we have analyzed whether a novel and endogenous weighting system can eliminate the benefits of gerrymandering. This weighting system sets each elected candidate's voting weight equal to the vote share received by the candidate's party within the entire jurisdiction divided by the representative share obtained by that party. Given these endogenous weights, it is straight-forward to show that a party's total weight in the legislature depends entirely on the vote share received by the party in the jurisdiction. As a result, in the baseline model of Gilligan and Matsusaka (1999), the weighting system guarantees that a gerrymanderer is unable to influence the policy outcomes of the legislature. Moreover, the resulting voting weight given to each party is proportionally responsive.

While this weighting system has a couple of appealing features, the drawbacks of the endogenous weights are large. The first drawback is implementability: if any proposed solution could effectively undermine the influence of a gerrymanderer, the party in power would be unlikely to implement it, as it reduces that party's ability to influence future elections and policies. Second, if the policy were enacted, realistic models imply that a gerrymanderer not only retains her influence, but she is better able to create policy bias.

References

- [1] Coate, Stephen and Knight, Brian. (2007). Socially Optimal Districting: A Theoretical and Empirical Exploration. *The Quarterly Journal of Economics* 122 (4): 1409-1471.
- [2] Friedman, John N. and Holden, Richard T. (2008). Optimal Gerrymandering: Sometimes Pack, but Never Crack. *The American Economic Review* 98 (1): 113-144.
- [3] Gelman, Andrew and King, Gary. (1990). Estimating the Electoral Consequences of Legislative Redistricting. *Journal of the American Statistical Association* 85 (410): 274-282.
- [4] Gelman, Andrew and King, Gary. (1994). Enhancing Democracy Through Legislative Redistricting. *The American Political Science Review* 88 (3): 541-559.
- [5] Gilligan, Thomas W. and Matsusaka, John G. (1999). Structural Constraints on Partisan Bias Under the Efficient Gerrymander. *Public Choice* 100: 65-84.
- [6] Gilligan, Thomas W. and Matsusaka, John G. (2006). Public Choice Principles of Redistricting. *Public Choice* 129: 381-398.
- [7] Gul, Faruk and Pesendorfer, Wolfgang. (2010). Strategic Redistricting. *The American Economic Review* 100 (4): 1616-1641.
- [8] King, Gary and Browning, Robert X. (1987). Democratic Representation and Partisan Bias in Congressional Elections. *The American Political Science Review* 81 (4): 1251-1273.
- [9] King, Gary. (1989). Representation through Legislative Redistricting: A Stochastic Model. *American Journal of Political Science* 33 (4): 787-824.
- [10] Kousser, J. M. (1996). Estimating the Partisan Consequences of Redistricting Plans-Simply. *Legislative Studies Quarterly* 21 (4): 521-541.
- [11] Shotts, Kenneth W. (2002). Gerrymandering, Legislative Composition, and National Policy Outcomes. *American Journal of Political Science* 46 (2): 398-414.