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In Search of a Fair Bet in the Lottery

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Abstract

Although state-operated lotto games have the worst average expected payoffs among common games of chance, because the jackpot can accumulate, the maximum expected payoff is potentially unlimited. It is possible, therefore, that lotto can exhibit a positive expected return. This paper examines 18,000 drawings in 34 American lotteries and finds approximately 1% of these drawings provided players with a fair bet. If it were possible for a bettor to purchase every possible combination, however, most lotteries commonly experience circumstances where such a purchase would provide a positive return with 11% of the drawings providing a fair bet to the player.

JEL Classification Codes: D81, H71, L83

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INTRODUCTION TO LOTTERY GAMES

"Lotto" is among the most popular games offered by state lottery associations accounting for 28% of total revenues for state-run U.S. lotteries in 1997. As of January, 2003, 38 states had lotteries, and every state association offered some version of a lotto game either through their own game or through a multi-state association such as the twenty-three member Multi-State Lottery Association (Powerball) or the nine state Mega Millions association.

Lotto games generally consist of an individual picking a set of five or six numbers from a group of approximately 35-55 choices. Winning numbers are then randomly selected at a weekly or bi-weekly drawing. A player whose ticket matches all of the winning numbers wins the jackpot prize while players matching some but not all of the winning numbers win smaller consolation prizes. In part, Lotto derives its popularity from the large jackpot prizes that can be won in this game. While lottery games such as instant tickets, numbers, or keno might offer top prizes ranging from $100 to $100,000, lotto games typically advertise jackpot prizes starting at $1 million or higher.

The jackpot prize is funded by allocating a percentage of ticket sales to the jackpot prize pool. If no ticket matches the winning numbers, the money in the fund is carried over into the next drawing and is added to the allocated funds from ticket sales in the next period. Because the jackpot prize fund is allowed to roll-over in this manner, the jackpot prize can become quite large if no one hits the jackpot in a large number of successive periods. Indeed, advertised jackpots exceeding $50 million are quite common, and occasionally lotto jackpots have been known to exceed $250 million.

Because lotto is one of the few games of chance where the expected return varies with

each drawing, these games have been widely studied in the academic literature, and the theory on buyer behavior and ticket payoffs is well-established. While numerous researchers have proposed the possibility that under specific conditions the lottery may present bettors with a "fair bet," that is a gamble with a positive expected return, their conclusions are generally based either solely on supposition or on the examination of just one or two lotto games. This short research note uses expected payoff functions developed in other research to answer the empirical question of whether lotteries actually are ever fair bets based on an extensive data set.

EXPECTED PAYOFFS FROM LOTTERIES

Testing whether lotto games present a fair bet requires an estimate of the expected return from the purchase of a lottery ticket. Several researchers have presented estimates of this expected return starting with Clotfelter and Cook [1989] and including DeBoer [1990], Krautmann and Ciecka [1992], Shapira and Venezia [1992], Scott and Gulley [1993, 1995], and Matheson [2001].

Since the price of a lotto ticket and the odds of winning remain fixed regardless of the size of the jackpot, it is natural to assume that the expected return of purchasing a lotto ticket will increase along with the size of the jackpot. The complicating factor, however, is that as the advertised jackpot grows, the number of ticket buyers typically increases as well. The increased number of ticket buyers increases the probability that the winning numbers will be shared by two or more tickets. Thus, the increase in expected return due to the increase in the size of the jackpot is tempered by the prospect of potentially having to share this larger jackpot among several winners. Following Matheson [2001], who presents the most detailed function, the

expected return expected return, ER_t, from the purchase of a single lottery ticket with randomly selected numbers is shown in equation (1).

$$(1) \qquad ER_t = [\sum^{i} w_i V_{it} + \frac{AV_{jt}}{dvr_t}(1 - e^{-B_t w_j})/B_t](1-\theta) + [\sum^{i} w_i + w_j]\theta\tau$$

where w_i is the probability of winning lower-tier prize i, V_{it} is the cash value of lower-tier prize i at time t, w_j is the probability of winning the jackpot prize, AV_{jt} is the advertised jackpot prize at time t, dvr_t is a divisor used to convert the advertised annuitized jackpot into a net present value, B_t is the number of other ticket buyers for the drawing in period t, θ is the tax rate, and τ is the price of a ticket.

It is fact that certain combinations of numbers (multiples of 7, birthdays, vertical or diagonal columns on the play slip, etc.) are more commonly played than other combinations, and therefore by playing rarer combinations a ticket buyer can earn an expected return above this average expected payout. The ability to earn above normal returns is limited by the amount to which the distribution of numbers played deviates from a uniform distribution. Since roughly 70% of all lotto tickets sold use computer generated numbers which can be reasonably assumed to follow a uniform distribution, any supernormal expected returns are limited to the deviation from uniformity by the 30% of tickets that are sold to players who select their own numbers. Furthermore, as lotto jackpots grow, the percentage of players selecting their own numbers falls, further reducing any ability of players to select advantageous numbers during periods of high expected returns. Still, the expected value in equation (1) should be seen as lower bound for the game. See Clotfelter and Cook [1989, p. 81], MacLean, et.al. [1992], or Thaler and Ziemba

[1988] for further discussion.

To test for fair bets in the lottery, data on jackpot size, ticket sales, and game format was collected from 34 state and multi-state lotto games representing over 18,000 individual drawings. For each drawing, the w_i's and w_j can be calculated in straight forward manner based on the game matrix of the specific lotto, and dvr_t can be closely estimated using prevailing interest rates and the annuity length of the jackpot prize. The value of the lower-tier prizes is also available by examining the specific game rules, and the expected jackpot is widely advertised by lottery associations prior to each drawing. A marginal tax rate of $\theta = 30\%$ was assumed.

A true representation of the *ex ante* expected value of purchasing a lottery ticket requires that the player be able to make an accurate estimation of the number of other ticket buyers. In order to facilitate the examination of a large number of lotto games, this paper will instead examine the *ex post* expected return from the purchase of a lotto ticket based on actual ticket sales rather than buyer forecasted ticket sales. While it is certainly true that the *ex post* and *ex ante* ticket sales (and hence *ex post* and *ex ante* returns) may differ from one another if players inaccurately estimate ticket sales, previous research has found that players can quite closely estimate ticket sales and do not generally make systematic forecasting errors [Gulley and Scott, 1995; Matheson and Grote, 2003]. Given these results, it can be said that the *ex ante* and *ex post* estimates approximately match one another on any individual drawing and that on average over many drawings will exactly match. For simply ascertaining the relative frequency of fair bets in the lottery, the *ex post* method gives a good approximation with a significant reduction in computational difficulty.

The results presented in Table 1 both confirm and counter the prevailing literature.

Overall, it is shown that fair bets are indeed rare occurrences with roughly 1% of drawings providing a player with a fair bet. On the other hand, the instances of fair bets may be significantly more common than previously believed. Half of the games studied showed at least one instance of a fair bet, and numerous games provided players with even odds on a relatively frequent basis. Several of the states exhibited even odds in 4% or more of the drawings.

It is also worthwhile to note that among the lotteries providing fair bets, several have maximum net expected payoffs well in excess of the price of the ticket with Indiana, Kansas, Kentucky and Missouri having a maximum expected gain of 40% or more and Oregon having a maximum expected return of over $2.20 on the purchase of a single one dollar ticket. Another fact that can be observed in Table 1 is the lotteries with positive maximum expected payoffs tend to be in smaller states. The eye-popping jackpots advertised in the Powerball and Big Game Lotteries as well as those in the bigger states such as New York, California, Texas, and Florida, attract large numbers of buyers diminishing the expected value of the ticket. As hypothesized by Forrest, *et al*, [2002], players seem to react to big jackpots rather than big expected returns.

EXPECTED PAYOFFS FROM THE "TRUMP TICKET"

It has been suggested that there may be conditions during which it may be profitable to corner a lottery game by purchasing every possible combination of numbers for a given drawing. Krautman and Ciecka [1993] and Matheson [2001] dub this strategy the "Trump Ticket." Calculating the expected payoffs requires some additional calculations. Assuming that other lottery players' decisions on whether to buy tickets remain constant regardless of whether another player buys the Trump Ticket, the purchase of a Trump Ticket does not affect the

probability of any single ticket winning the jackpot nor does it change the expected number of winning tickets among the other buyers in the particular drawing. The purchase does, however, increase the size of the jackpot that the jackpot winner(s) receives. Since the purchase of the Trump Ticket necessitates a large purchase of tickets, if a specific portion of ticket sales is allocated to the jackpot prize pool, as in most games, the purchase of the Trump Ticket will cause a significant increase in the size of the jackpot. Mathematically, $AV_{jt}^{TT} = AV_{jt} + \tau \alpha_j dvr_t / w_j$ where AV_{jt}^{TT} is the advertised jackpot after the purchase of the Trump Ticket and α_j is the percentage of gross sales allocated to the jackpot pool. Since all number combinations are chosen under a Trump Ticket strategy, it is not necessary to assume that other players' number selections are uniformly distributed.

The issue of taxation again must be considered. As with the purchase of a single ticket, any winnings are fully taxable at the rate θ, but the Trump Ticket purchaser may deduct the cost of the tickets purchased to the extent of any winnings. If the purchaser's winnings exceed the cost of the Trump Ticket then the winnings less the cost of the Trump Ticket are taxable. If the purchaser's winnings are less than the cost of the Trump Ticket, then the full cost of the Trump Ticket is not deductible, but the purchaser will not have to pay taxes on any of the winnings, either.

Table 2 shows the maximum expected return per dollar played for both a single ticket and a Trump Ticket purchase for every lotto game as well as the number of Trump Ticket drawings providing a fair bet. In comparing Tables 1 and 2, the first obvious conclusion is that Trump Ticket purchases are more often associated with positive expected returns than are single ticket purchases. As noted by Matheson [2001], the purchase of a Trump Ticket always has a higher

expected return per dollar played than the purchase of a single ticket for two reasons. First, the purchase of the Trump Ticket increases the size of the jackpot without changing the expected number of other players matching the jackpot ticket leading to a higher expected payout from the grand prize. Second, because the purchase of the Trump ticket guarantees at least a share in the winning jackpot (as well as lower tier prizes), the purchaser of the Trump Ticket has a much higher chance of being able to deduct the price of the tickets from applicable taxes than the purchaser of a single ticket. Therefore, a significantly greater number of the lotteries studied provide opportunities for positive expected returns for the Trump Ticket purchaser than for the single ticket purchaser. With only one exception, each lottery examined shows at least one instance of the Trump Ticket providing greater than even odds.

The other startling aspect of Table 2 is simply the extraordinarily high number of times that the Trump Ticket presents a fair bet. Overall, 11% of the drawings examined provided an even odds bet for the purchase of the Trump Ticket with one-third of the games presenting an fair bet during at least 20% of draws. The size of the potential winnings is also surprising with many games offering an after-tax expected rate of return of over 50% at their highest point.

CONCLUSIONS AND POLICY RECOMMENDATIONS

The results presented in this paper suggest that it is not only theoretically possible for lotteries to exhibit periods where the purchase of a single lottery ticket has a positive net present value, it is in fact a regular, though uncommon, occurrence for lotteries, especially in smaller states lotteries, to exhibit this trait. Since the presence of a fair bet in the purchase of a single lottery ticket represents a violation of efficient markets, lottery associations where fair bets

routinely occur should be able to increase ticket sales in the presence of these higher expected returns through public education and better advertising of high jackpots. Gamblers should also take note that while the huge jackpots associated with the large multi-state games attract the most media attention and the frenzied buying, the best returns to players occur in the smaller games where relatively large jackpots do not spur "lotto fever" and the associated reduction in the expected values.

In addition, it is extremely common that the purchase of the Trump Ticket, i.e. the purchase of all available combinations, would provide a fair bet to the buyer. The fact that investment consortiums do not routinely attempt to corner lotto jackpots is likely due to the transaction costs associated with the purchase of every number combination. One recommendation would be that lottery associations consider allowing for the direct purchase of a Trump Ticket by investment consortiums although the one-time ticket sale gains must be balanced against the possible loss of trust in the lottery by the public, who may feel that such direct purchase is akin to "fixing" the lottery. In addition, if ticket sales are fueled by stories of the "regular guy" hitting it big, it is likely that stories of rich investment consortiums getting even richer through taking advantage of such a direct purchase may depress sales further. Still, such a policy may be intriguing to lottery associations, which have generally experienced flat sales over the past several years.

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TABLE 1
Expected Returns for Single Ticket purchase

Lottery	Dates of Data	Highest Observed Jackpot	Max. Expected Return (per $1.00 played)	# of Draws	# of Positive Draws	% Positive Draws
Multi-state “Powerball”	4/22/92 - 1/15/03	$315 million	$0.727	1,121	0	0%
Multi-state “Big Game”	9/06/96 - 5/04/99	$190 million	$0.776	215	0	0%
Tri-State “Megabucks”	3/12/97 - 5/29/99	$8.2 million	$0.719	214	0	0%
Tri-State “Win Cash”	9/12/97 - 5/28/99	$2.33 million	$0.973	179	0	0%
Tri-West “Lotto”	2/04/95 - 1/31/98	$1.63 million	$1.067	307	2	0.7%
Multi-state “Wild Card”	2/04/98 - 7/28/01	$2.06 million	$0.681	364	0	0%
Arizona “Lotto”	11/28/98 - 5/22/99	$10.1 million	$0.921	51	0	0%
California “Super Lotto”	10/18/86 - 1/19/02	$141 million	$0.753	1,544	0	0%
Colorado “Lotto”	9/14/90 - 7/28/01	$27 million	$0.977	1,150	0	0%
Connecticut “Lotto”	9/20/94 - 8/07/01	$26 million	$1.251	719	10	1.4%
Delaware “All Cash”	10/27/98 - 5/18/99	$1.13 million	$0.888	88	0	0%
Florida “Lotto”	5/07/88 - 7/28/01	$106.5 million	$0.945	783	0	0%
Georgia “Lotto”	8/31/96 - 8/04/01	$30.4 million	$1.027	258	1	0.4%
Illinois	4/14/99 - 8/01/01	$33 million	$1.253	241	6	2.5%
Indiana	9/03/94 - 8/01/01	$42 million	$1.292	542	9	1.7%
Kansas “Cash”	8/18/96 - 5/12/99	$2.00 million	$1.565	428	21	4.9%
Kentucky “Lotto”	3/01/95 - 7/28/01	$20 million	$1.444	670	29	4.3%
Louisiana	4/19/98 - 5/22/99	$2.05 million	$0.660	114	0	0%
Maryland	1/03/98 - 7/14/99	$18.5 million	$1.144	160	5	3.1%
Mass. “Megabucks”	11/05/97 - 8/11/01	$14.3 million	$1.340	394	21	5.3%
Mass. “Millions”	11/06/97 - 8/13/01	$30.6 million	$1.145	394	6	1.5%
Michigan “Lotto”	9/04/96 - 7/28/01	$40 million	$1.159	497	10	2.0%
Minnesota “Gopher 5"	5/24/91 - 7/24/01	$1.40 million	$0.918	1,062	0	0%
Missouri “Lotto”	1/03/96 - 6/30/01	$11.6 million	$1.546	459	22	4.8%
New Jersey	7/03/95 - 4/05/99	$35 million	$1.086	393	1	0.3%
New York	4/14/99 - 8/01/01	$45 million	$0.691	375	0	0%
Ohio “Super Lotto”	1/12/91 - 7/28/01	$54 million	$1.004	1,099	1	0.1%
Oregon “Lotto”	4/19/95 - 5/19/01	$18 million	$2.204	636	32	5.0%
Pennsylvania “Pick 6"	9/12/98 - 8/04/01	$73 million	$0.843	303	0	0%
South Dakota “Cash”	7/03/96 - 8/11/01	$0.34 million	$0.884	530	0	0%
Texas “Lotto”	11/14/92 - 1/15/03	$85 million	$0.969	1,061	0	0%
Virginia “Lotto”	1/27/90 - 5/05/99	$28 million	$1.168	929	6	0.7%
Washington	1/01/97 - 5/26/99	$24 million	$1.042	251	2	0.8%
Wisconsin	6/20/92 - 5/15/99	$16.5 million	$0.812	721	0	0%
Total				18,252	184	1.0%

TABLE 2
Expected returns for the Trump Ticket purchase

Lottery	Max. Expected Return: Single Ticket	Max. Expected Return: Trump Ticket	# of Draws	# of Positive Draws	% Positive Draws
Powerball	$0.727	$1.036	1,121	10	0.9%
Big Game	$0.776	$1.120	215	2	0.9%
Tri- Megabucks	$0.719	$1.114	214	3	1.4%
Tri- Win Cash	$0.973	$1.443	179	33	18.4%
Tri-West	$1.067	$1.590	307	67	21.8%
Wild Card	$0.681	$1.234	364	21	5.8%
Arizona	$0.921	$1.434	51	14	27.5%
California	$0.753	$1.109	1,544	7	0.5%
Colorado	$0.977	$1.397	1,150	91	7.9%
Connecticut	$1.251	$1.767	719	202	28.1%
Delaware	$0.888	$1.438	88	30	34.1%
Florida	$0.945	$1.321	783	23	2.9%
Georgia	$1.027	$1.368	258	28	10.9%
Illinois	$1.257	$1.745	241	43	17.8%
Indiana	$1.292	$1.812	542	86	15.9%
Kansas	$1.565	$2.055	428	93	21.7%
Kentucky	$1.444	$2.014	670	227	33.9%
Louisiana	$0.660	$0.982	114	0	0%
Maryland	$1.144	$1.545	160	45	28.1%
Mass Mega	$1.340	$1.764	394	87	22.1%
Mass Millions	$1.145	$1.630	394	145	36.8%
Michigan	$1.159	$1.488	497	60	12.1%
Minnesota	$0.918	$1.338	1,062	76	7.2%
Missouri	$1.546	$1.911	459	102	22.2%
New Jersey	$1.086	$1.531	393	27	6.9%
New York	$0.691	$1.043	375	3	0.8%
Ohio	$1.004	$1.281	1,099	48	4.4%
Oregon	$2.204	$2.498	636	96	15.1%
Pennsylvania	$0.853	$1.173	303	27	8.9%
South Dakota	$0.884	$1.330	530	34	6.4%
Texas	$0.969	$1.189	1,061	52	4.9%
Virginia	$1.168	$1.670	929	127	13.7%
Washington	$1.042	$1.305	251	19	7.6%
Wisconsin	$0.812	$1.360	721	84	11.7%
Total			18,252	2,012	11.0%