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Abstract

This paper analyzes the impact of consumer uncertainty on optimal fiscal policy in a model with capital. The consumers lack confidence about the probability model that characterizes the stochastic environment and so apply a max-min operator to their optimization problem. An altruistic fiscal authority does not face this Knightian uncertainty. It is shown analytically that the government, in responding to consumer uncertainty, no longer sets the expected capital tax rate exactly equal to zero, as is the case in the full-confidence benchmark model. However, our numerical results indicate that the government does not diverge far from this value. Even though the capital income tax rate is close to zero in expectation, consumer uncertainty leads the altruistic government to implement a more volatile capital tax rate across states. In doing so, the government relies more heavily on the capital tax and, consequently, less heavily on the labor income tax to finance the shock to public spending.

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Abstract

This paper analyzes the impact of consumer uncertainty on optimal fiscal policy in a model with capital. The consumers lack confidence about the probability model that characterizes the stochastic environment and so apply a max-min operator to their optimization problem. An altruistic fiscal authority does not face this Knightian uncertainty. It is shown analytically that the government, in responding to consumer uncertainty, no longer sets the expected capital tax rate exactly equal to zero, as is the case in the full-confidence benchmark model. However, our numerical results indicate that the government does not diverge far from this value. Even though the capital income tax rate is close to zero in expectation, consumer uncertainty leads the altruistic government to implement a more volatile capital tax rate across states. In doing so, the government relies more heavily on the capital tax and, consequently, less heavily on the labor income tax to finance the shock to public spending.

1   Introduction:

In the typical public finance model with rational expectations, fiscal policy influences consumer behavior through two channels. First, policy can have a contemporaneous effect. By adjusting a labor income tax, for example, the government alters the consumers' incentives to supply labor in that period. The second channel is through the consumers' expectations. By committing to future policy, the government shapes the consumers' beliefs about the possible paths of the endogenous variables, such as asset returns and the marginal utility of consumption. In doing so, future policies affect the consumers' behavior in earlier periods. The assumption of rational expectations helps facilitate this second, inter-temporal channel, enabling the consumers to correctly forecast both the state-contingent values of the endogenous variables and the probability model over these variables.

Rational expectations, though, might exaggerate the ability of consumers to understand the stochastic equilibrium. This exaggeration could be costly in that it might mean that the typical fiscal policy model

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overemphasizes how precisely consumers respond to future policy commitments of the government. If instead consumers face uncertainty about the economy's true probability model, their expectations and behavior might be quite different than those predicted in a rational expectations model. As a consequence, the fiscal authority might find it optimal to implement a different set of fiscal policies knowing that the consumers face model uncertainty. Therefore, consumer uncertainty might lead to substantial changes in both consumer behavior and optimal fiscal policy relative to a rational expectations framework.

Karantounias, Hansen, and Sargent (2009) and Svec (2011) are two examples that introduce consumer uncertainty in an optimal fiscal policy model. In these models without capital, the authors show that the consumers' uncertainty does indeed alter the government's policy decisions. This is because fiscal policy must mitigate the welfare costs associated with both linear taxes and consumer uncertainty. Depending on the specific type of altruism exhibited by the planner, the optimal policy involves either more or less reliance on the labor income tax to finance public spending than is optimal under the baseline model in which consumers do not face model uncertainty.

Although these results are suggestive, the impact of consumer uncertainty on optimal fiscal policy should be most salient in a model with capital, as the consumers' expectations are of primary importance in the design of optimal policy. A prime example that highlights this importance in a rational expectations model is Chari, Christiano, and Kehoe (1994). In this model, consumers supply labor and can invest in either capital or one-period, state-contingent government debt. A Ramsey planner sets a labor tax and a capital income tax to maximize the consumers' expected utility. As the authors show, the planner finds it optimal to structure the state-contingent capital income tax rates so that the consumers expect a zero percent capital income tax rate. This policy choice encourages the consumers to invest in capital as they would in the first-best solution. Thus, in this rational expectations model, the government forgoes collecting any tax revenue from capital income on average in order to impart the correct beliefs to consumers.

But, if consumers were uncertain as to the economy's true probability model and so behaved according to a different expectation, they might choose a different investment profile than would be optimal under the assumption of rational expectations. Further, in responding to this uncertainty, the planner might alter its policies in order to influence the consumers' behavior under uncertainty. Consequently, the stark and powerful policy prescription that the government should optimally implement an expected capital tax rate equal to zero might break down under consumer uncertainty. For this reason, it is particularly critical to understand the implications of consumer uncertainty in a fiscal policy model with capital.

The current paper fills this role by introducing consumer uncertainty into the neoclassical growth model of Chari, Christiano, and Kehoe (1994). To formalize this uncertainty and the consumers' resulting behavior, this paper follows Hansen and Sargent (2001, 2005, 2007) and the robust control literature.

In this approach, consumers are unsure which probability model characterizes the economy. Instead, they believe that the true probability model lies somewhere within a range of alternative probability models. Each alternative model is represented as a martingale perturbing the approximating probability model. With this type of uncertainty, the robust control literature assumes that the consumers optimize according to max-min preferences, choosing the allocation that maximizes their expected utility, where the expectation is taken with respect to the probability model that minimizes their expected utility. The resulting allocation is labeled the robustly optimal allocation, and the worst-case probability model is labeled the consumers' subjective probability model. This behavior helps ensure that the consumers' utility never falls too far, regardless of which probability model happens to be correct.

Although it is assumed that the consumers are uncertain as to the correct probability model, the opposite assumption is made for the fiscal authority: the government is fully confident that the approximating probability model truly characterizes the stochastic environment. To be clear, the consumers and the government are both endowed with the same approximating model. This approximating model specifies the probability model associated with the exogenous and endogenous variables. However, the consumers doubt the accuracy of this model, while the government trusts that it correctly describes the economy's probability model.

Critically, this confidence dichotomy reveals a number of possible objective functions for an altruistic government. These objective functions differ as to which expectation they use in calculating the consumers' expected utility. That is, the government could optimize with respect to the approximating probability model or it could optimize according to any one of the alternative probability models that the consumers believe could describe the economy, including the subjective probability model. As the consumers distrust the government's confidence in the approximating probability model, it is not clear which model an altruistic government should use in its optimization problem.

Given this multiplicity of possible objective functions, I will assume in this paper that the fiscal authority maximizes the consumers' expected utility under the consumers' own subjective expectation.[1] This choice of objective function allows for a one-step deviation from the rational expectations framework, since both the consumers and the planner optimize with respect to the same expectation. Just as important, the consumers would likely prefer this type of government because its objective function is better aligned with their own preferences than one that optimized according to the approximating probability model.

With this setup, the optimal policy implemented by the fiscal authority involves one period of transition. During that period, the government subsidizes labor with a negative tax on labor income and implements

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[1]In a follow-up paper, I assume that the fiscal authority maximizes the consumers' expected utility under the approximating model.

a large tax on capital income, as in Chari, Christiano, and Kehoe (1994). From that period forward, there are three main properties of the time-invariant optimal policies. First, it can be shown analytically that, under one condition, the expected capital tax rate is non-zero, breaking the rational expectations result. To derive the magnitude and direction of this deviation from zero, I turn to my numerical implementation of the model. It is found quantitatively that the government chooses to subsidize the consumers' capital income, on average, at a modest rate. This subsidy is important in mitigating the pessimism associated with consumer uncertainty.

Second, relative to the full-confidence benchmark, consumer uncertainty leads the government to increase the covariance of both a private assets tax and the ex-post capital income tax with respect to public spending. An implication of this increase is that the government relies more heavily on these capital income taxes to finance the deviation of spending from its mean. During periods of high spending, for example, the government pays for the rise in expenditure largely through a combination of lowering the return on public debt and raising the capital income tax rate. The third policy consequence of consumer uncertainty is that the government should smoothe the fluctuations in the labor income tax rate across states. In fact, if consumers face a sufficiently high degree of uncertainty, the government implements a constant labor tax across states.

The current paper fits into a larger strand of the recent literature that analyzes how model uncertainty alters the policy conclusions derived from rational expectations models. Generally, this literature has focused on planner uncertainty within a monetary policy framework; examples include Dennis (2010), Dennis, Leitemo, and Soderstrom (2009), Hansen and Sargent (2008), Leitemo and Soderstrom (2008), Levin and Williams (2003), Onatski and Stock (2002), and Walsh (2004). Woodford (2010) modifies the type of uncertainty considered by assuming that the central bank is uncertain of the expectations held by firms, but not uncertain about the stochastic environment. Thus, in addition to examining fiscal policy rather than monetary policy, the current analysis differs from most of the literature by examining the policy implications of consumer uncertainty rather than the planner's uncertainty. Finally, this paper is novel in that, to the best of my knowledge, it is the first to analyze optimal capital income tax rates in a model with consumer uncertainty.

The outline of the paper is as follows. Section 2 describes the economic environment and characterizes the type of uncertainty faced by the consumers. The optimization problem of the consumers is also formulated. Section 3 discusses the planner's optimization problem. In addition, this section includes the analytical result that the fiscal authority no longer sets the ex-ante capital income tax equal to zero. Section 4 examines the numerical results, and Section 5 concludes.

2 The Economy:

Time is discrete in this infinite-horizon production economy. There are three types of agents: a government, an infinite number of identical consumers, and firms. The only source of randomness in the model is a shock to government spending. This shock can take on a finite number of values. Let $g^t = (g_0, ..., g_t)$ represent the history of the spending shock up to and including period t, where the probability of each history is $\pi(g^t)$. In period 0, government spending is known to be g_0 with probability 1. The government finances this expenditure through either taxes or debt, b_t. The government has access to a labor income tax, τ_t, and a capital income tax, Ω_t. Both are restricted to be proportional taxes. Government debt has a state-contingent return, $R_{b,t}$, and matures in one period. The period budget constraint of the government is

$$b_t = R_{b,t}b_{t-1} + g_t - \tau_t w_t l_t - \Omega_t [r_t - \delta] k_{t-1} \tag{1}$$

Note that the capital income tax applies to the after-depreciation return on capital, where δ is the depreciation rate.

Each consumer's wealth is composed of three components: after-tax labor income, after-tax capital income, and a return on debt held from the previous period. Out of this wealth, the consumer can choose to consume, buy capital, or save in the debt market. In each period, the consumer also chooses how much labor to supply. The period budget constraint for the consumer is

$$c_t + k_t + b_t \leq (1 - \tau_t) w_t l_t + R_{k,t} k_{t-1} + R_{b,t} b_{t-1} \tag{2}$$

where $R_{k,t} = 1 + (1 - \Omega_t)(r_t - \delta)$ is the gross, after-tax return on capital.

A constant returns to scale production function, $F(k_{t-1}, l_t)$, transforms labor and capital into output. This production function satisfies the Inada conditions. The resulting output can be used for private consumption c_t, public consumption g_t, or investment $k_t - (1 - \delta) k_{t-1}$. The economy-wide resource constraint is therefore

$$c_t + k_t + g_t = F(k_{t-1}, l_t) + (1 - \delta) k_{t-1} \tag{3}$$

Competitive firms ensure that the returns on labor and capital equal their respective marginal products:

$$w_t = F_l(k_{t-1}, l_t) \tag{4}$$

and

$$r_t = F_k(k_{t-1}, l_t) \tag{5}$$

2.1 The Consumers' Model Uncertainty:

The consumers are endowed with an approximating probability model that specifies a probability measure over the paths of the exogenous and endogenous variables. Unlike in Chari, Christiano, and Kehoe (1994), the consumers are uncertain whether this approximating model correctly characterizes the equilibrium. Instead, they worry that other probability measures could potentially describe the stochastic nature of the economy. To ensure that these alternative models conform to some degree with the approximating model, restrictions must be placed on what types of alternative models are allowed.

Following Hansen and Sargent (2005, 2006), it is assumed that each member of the set of alternative probability distributions is absolutely continuous with respect to the approximating model. This requirement implies that the consumer only fears models that correctly put no weight on zero probability events. That is, if fiscal policy implies that a certain event will never occur, the consumers must also believe that this is true. Thus, an alternative model can place a different weight on a history relative to the approximating model as long as the probability of that history under the approximating model is between zero and one. More specifically, the assumption placed on the alternative models is that they must be absolutely continuous over finite time intervals. This implies that the alternative models entertained by the consumers cannot be rejected with a finite amount of data, even though they could be rejected with an infinite data set.

With the assumption of absolute continuity, the Radon-Nikodym Theorem indicates that there exists a measurable function, M_t, such that the subjective expectation of a random variable, X_t, can be rewritten in terms of the approximating probability model:

$$\widetilde{E}[X_t] = E[M_t X_t]$$

where $E[M_t] = 1$ and $\widetilde{E}$ is the subjective expectations operator. This is important, as it allows me to recast consumer uncertainty. Earlier, the consumers were described as being uncertain about the probability model that characterizes the paths of the exogenous and endogenous variables; now, the consumers can be viewed as understanding the correct mapping from states of the world to equilibrium outcome, even though they may not place the correct probability on each state.

By defining an additional term, one can begin to measure the distance between an alternative probability model and the approximating probability model. Let the incremental probability distortion be

$$m_{t+1} = \frac{M_{t+1}}{M_t}, \forall M_t > 0$$

and $m_{t+1} = 1$ otherwise. This incremental distortion must satisfy $E_t m_{t+1} = 1$, implying that the probability distortion M_t is a martingale. This restriction guarantees that the alternative probability measures are legitimate probability models. With this definition, the one-period distance between the

alternative and approximating models is measured by relative entropy:

$$\epsilon_t \left(m_{t+1} \right) \equiv E_t m_{t+1} \log m_{t+1}$$

This measure is convex and grounded, attaining its minimum when $m_{t+1} = 1, \forall g_{t+1}$.

Each period's relative entropy can be aggregated and discounted to form a measure of the total distortion relative to the approximating model:

$$E_0 \sum_{t=0}^{\infty} \beta^t M_t \epsilon_t \left(m_{t+1} \right)$$

This distance measure is used in the multiplier preferences of Hansen and Sargent (2006). The multiplier preferences characterize how the consumers rank their allocations. Given these preferences, the consumers choose the allocation that maximizes the following criteria:

$$\min_{m_{t+1}, M_{t+1}} \sum_{t=0}^{\infty} \sum_{g^t} \beta^t \pi \left(g^t \right) M_t \left[u \left(c_t, l_t \right) + \beta \theta \epsilon_t \left(m_{t+1} \right) \right]$$

where $u \left(c, l \right)$ is increasing in consumption, decreasing in labor, and strictly concave.

The coefficient $\theta > 0$ is a penalty parameter that indexes the degree to which consumers are uncertain about the probability measure. A small θ implies that the consumers are not penalized too harshly for distorting their probability model away from the approximating model. The min operator then yields incremental probability distortions that diverge greatly from one. The resulting probabilities $\{ \pi \left(g^t \right) M_t \}$ are distant from the approximating model. Thus, a small θ indicates that consumers are very unsure about the approximating model and so fear a large set of alternative models. A larger θ means that the consumers face a sizable penalty for distorting their probability model away from the approximating model. As a result, the min operator yields incremental distortions close to one, implying that the worst-case alternative model is close to the approximating model. Thus, a large θ signifies that the consumers have more confidence about the underlying measure and so fear only a small set of alternative models. As $\theta \rightarrow \infty$, this model collapses to the rational expectations framework of Chari, Christiano, and Kehoe (1994).

2.2 The Consumer's Problem:

With this formalism, the consumer's problem can be written recursively using the value function $V \left(b_-, k_-, g, A \right)$:

$$V \left(b_-, k_-, g, A \right) = \max_{c,l,b,k} \min_{m'} \left\{ \begin{array}{c} u \left(c, l \right) + \beta \sum_{g'} \pi \left(g' \mid g \right) \left[m' V \left(b, k, g', A' \right) + \theta m' \log m' \right] \\ \\ - \lambda \left[c + k + b - \left(1 - \tau \right) wl - R_b b_- - R_k k_- \right] \\ \\ - \beta \theta \Psi \left[\sum_{g'} \pi \left(g' \mid g \right) m' - 1 \right] \end{array} \right\}$$

where A represents the set of aggregate state variables that the consumers must track in order to forecast fiscal policy in all histories. This set of state variables comes from the government's optimization problem. The consumer believes that her decisions cannot affect the movements of these aggregate state variables. In addition to the period budget constraint, the consumer faces the legitimacy constraint, $\sum_{g'} \pi(g' \mid g) m' = 1$, described above.

Solving the consumer's Bellman equation for the robustly optimal allocation is a two-stage process. In the inner minimization stage, the consumer fears that, for a given allocation, the worst-case probability model over the government spending shocks will occur. The solution that results from this minimization is the consumer's subjective expectation. The outer maximization stage determines the allocation that maximizes the consumers' expected utility, taking into account the endogenous tilting of the consumers' expectation. The solution from this stage is the consumer's robustly optimal allocation.

2.2.1 The Inner Minimization Stage:

As indicated above, the minimization stage yields the subjective probability model that minimizes the consumer's expected utility for a given allocation. The state-contingent probability distortion, which balances the marginal benefit of lowering the consumer's expected utility with the marginal cost of the convex penalty term, solves the following equation:

$$V(b, k, g', A') + \theta(1 + \log m') - \theta\Psi = 0$$

Combining this first order condition with the legitimacy constraint, the optimal distortion is

$$m' = \frac{\exp\left(\frac{-V(b,k,g',A')}{\theta}\right)}{\sum_{g'} \pi(g' \mid g) \exp\left(\frac{-V(b,k,g',A')}{\theta}\right)} \tag{6}$$

This equation describes the consumer's worst-case, state-contingent incremental probability distortion. The magnitude and direction of this distortion depend upon the consumer's subjective welfare, V, in each state in period $t+1$. To better understand this function, consider a two-state government spending process. Suppose that the equilibrium allocation yields a high subjective welfare in state A and a low subjective welfare in state B. Plugging these values into (6), we see that $m_A < 1$ and $m_B > 1$. These distortions imply that consumers fear that the likelihood of state A is small and that the likelihood of state B is large relative to the approximating model.

The degree to which these multiplicative distortions diverge from unity depends upon θ and the difference between V_H and V_L. All else equal, a large θ decreases the probability distortion in all states in period $t+1$, meaning that $\{m_{t+1}\}$ remains closer to one. A small θ, conversely, implies that the

probability distortions are further away from one. Also, all else equal, as the difference between V_H and V_L grows, the consumer's alternative model is increasingly far from her approximating model.

2.2.2 The Outer Maximization Stage:

In the maximization stage, the consumer chooses an allocation that performs well even if the worst-case shock process truly characterizes government spending. To find this allocation, I have incorporated the subjective probability model that is derived in the minimization stage into the consumer's optimization problem. The resulting Bellman equation is

$$
V\left(b_-, k_-, g, A\right) = \max_{c,l,b,k} \left\{ \begin{array}{c} u\left(c, l\right) - \beta\theta \log \sum_{g'} \pi\left(g' \mid g\right) \exp\left(\frac{-V\left(b,k,g',A'\right)}{\theta}\right) \\ -\lambda\left[c + k + b - \left(1 - \tau\right)wl - R_b b_- - R_k k_-\right] \end{array} \right\}
$$

This equation highlights the fact that the consumer does not weight her future welfare as she would if she were fully confident in the approximating probability model. Rather, the allocation alters the consumer's future subjective welfare, which in turn influences the endogenous probability distortion.

As is standard in fiscal policy models in which the government must set linear taxes, the intra-temporal condition between consumption and labor is

$$
-\frac{u_l\left(c, l\right)}{u_c\left(c, l\right)} = \left(1 - \tau\right)w \tag{7}
$$

This equation links the marginal disutility of labor with the marginal benefit of raising consumption through increased labor supply. The linear labor tax distorts the optimal tradeoff away from the first-best: $-\frac{u_l(c,l)}{u_c(c,l)w} = 1$.

The two inter-temporal conditions are

$$
1 = \beta \sum_{g'} \pi\left(g' \mid g\right) m' \frac{u_c\left(c', l'\right)}{u_c\left(c, l\right)} R_b' \tag{8}
$$

$$
1 = \beta \sum_{g'} \pi\left(g' \mid g\right) m' \frac{u_c\left(c', l'\right)}{u_c\left(c, l\right)} R_k' \tag{9}
$$

These equations balance the marginal utility of increasing consumption today with the expected marginal utility from saving that additional unit in the debt or capital markets. Since the consumer faces model uncertainty, the conditional expectation within these equations is taken with respect to the subjective probability model.

The envelope conditions are

$$
V_b\left(b_-, k_-, g; A\right) = \lambda R_b
$$

$$
V_k\left(b_-, k_-, g; A\right) = \lambda R_k
$$

Definition 1 *Given an initial allocation $\{b_{-1}, k_{-1}\}$, an initial policy value Ω_0, and an initial return on debt $R_{b,0}$, a competitive equilibrium is a history-dependent allocation $\{c_t, l_t, b_t, k_t\}_{t=0}^{\infty}$, probability distortions $\{m_{t+1}, M_{t+1}\}_{t=0}^{\infty}$, prices $\{r_t, w_t\}_{t=0}^{\infty}$, returns $\{R_{k,t+1}, R_{b,t+1}\}_{t=0}^{\infty}$, and fiscal policies $\{\tau_t, \Omega_t\}_{t=0}^{\infty}$ such that*

1. *The probability distortion solves the consumer's inner minimization problem*

2. *The allocation solves the consumer's outer maximization problem, and*

3. *The allocation is feasible, satisfying* (3).

3 The Government's Problem:

This section considers the policy problem of the government. It is assumed that the government has access to a commitment technology with which it is able to bind itself to a sequence of policies chosen at $t = 0$. Unlike the consumers, the government is fully confident that the approximating probability model accurately describes the government spending process.

As the definition of the competitive equilibrium makes clear, there are a continuum of possible competitive equilibria, each indexed by a fiscal policy $\{\tau_t, \Omega_t\}_{t=0}^{\infty}$. The outcome, then, depends upon the objective of the fiscal authority. For the purposes of this paper, I assume that the planner maximizes the consumers expected utility under the consumers' subjective probability model. This decision implies that the government optimizes with respect to the same probability model as the consumers. Given that the consumers are uncertain about the true probability model and the government has no more information as to the true probability model than do the consumers (instead, the government is just more confident in the approximating model), the consumers might prefer this type of government to one that optimized according to a different probability model.

With this choice of planner preferences, the Ramsey outcome is the competitive equilibrium that attains the maximum. In formulating the Ramsey problem, I will follow the primal approach in which the government chooses the consumers' allocation and probability distortions. With these values, I will then back out what fiscal policies implement this competitive equilibrium.

Proposition 1 *The allocation and distortions in a Ramsey outcome solve the following problem:*

$$\max_{c_t, l_t, V_t, k_t, M_t, m_{t+1}} \sum_{t=0}^{\infty} \sum_{g^t} \beta^t \pi\left(g^t\right) M_t u\left(c_t, l_t\right)$$

subject to

$$\sum_{t=0}^{\infty} \sum_{g^t} \beta^t \pi\left(g^t\right) M_t \left[u_c\left(c_t, l_t\right) c_t + u_l\left(c_t, l_t\right) l_t\right] = u_c\left(c_0, l_0\right) \left[R_{b0} b_{-1} + R_{k0} k_{-1}\right] \tag{10}$$

$$m_{t+1} = \frac{\exp\left(\frac{-V_{t+1}}{\theta}\right)}{\sum\limits_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) \exp\left(\frac{-V_{t+1}}{\theta}\right)} \tag{11}$$

$$V_t = u\left(c_t, l_t\right) + \beta \sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) \left\{m_{t+1} V_{t+1} + \theta m_{t+1} \ln m_{t+1}\right\} \tag{12}$$

$$M_{t+1} = m_{t+1} M_t \tag{13}$$

$$c_t + g_t + k_t = F\left(k_{t-1}, l_t, g_t\right) + (1 - \delta) k_{t-1} \tag{14}$$

Proof. When setting its policy, the government is restricted in the set of feasible allocations that it can achieve by the competitive equilibrium constraints. The claim is that those restrictions are summarized by the constraints $(10) - (14)$. To demonstrate this, I will first show that any allocation and probability distortion that satisfies the competitive equilibria constraints must also satisfy $(10) - (14)$. Multiply (2) by $\beta^t \pi\left(g^t\right) M\left(g^t\right) \lambda\left(g^t\right)$ and sum over t and g^t. Plugging in $(7) - (9)$ and using the two transversality conditions

$$\lim_{T \to \infty} \beta^T M_T \lambda_T b_T = 0$$
$$\lim_{T \to \infty} \beta^T M_T \lambda_T k_T = 0$$

reveals the constraint (10). The constraint (11) follows directly from the optimality condition in the inner minimization, (13) comes from the definition of m_{t+1}, and (12) is the representative consumer's Bellman equation. Finally, (14) is the resource constraint which ensures feasibility. Thus, $(10) - (14)$ are necessary conditions that the Ramsey outcome must solve. Going in the other direction, given an allocation and distortions that satisfy $(10) - (14)$, policies and prices can be determined from $(1) - (5)$ and the consumer's first order conditions. ∎

The first constraint in the planner's problem is the implementability constraint. This constraint differs from its rational expectations counterpart in that the planner must account for the consumers' probability distortion at each date t. This is accomplished by the multiplicative term, M_t. In order to incorporate how policy affects this distortion, the planner must keep track of how that distortion is set and how it is updated across time and state. This information is contained in the next three constraints. The final constraint is the resource constraint.

The proposition above describes the robustly optimal allocation and distortions that achieve the Ramsey outcome. The bond holdings in history g^r that support this competitive equilibrium are described by

$$b_r = \frac{\sum\limits_{t=r+1}^{\infty} \sum\limits_{g^t} \beta^{t-r} \pi\left(g^t \mid g^r\right) M_t \left[u_c\left(c_t, l_t\right) c_t + u_l\left(c_t, l_t\right) l_t\right]}{M_r U c\left(c_r, l_r\right)} - k_r \tag{15}$$

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This value is pinned down using the future, state-contingent values of consumption, labor supply, capital, and probability distortions.

It is known that the government has the incentive to finance its public spending by raising taxes on the inelastic goods of capital and debt at $t = 0$. To prevent this outcome, I assume exogenous values for the initial capital tax, Ω_0, and return on debt $R_{b,0}$.

3.1 Sequential Formulation of Ramsey Problem:

With this setup, I now formulate the government's sequential problem:

$$
\mathcal{L} = \sum_{t=0}^{\infty} \sum_{g^t} \beta^t \pi\left(g^t\right) \left\{ \begin{array}{c} M_t u\left(c_t, l_t\right) + \xi M_t \left[u_c\left(c_t, l_t\right) c_t + u_l\left(c_t, l_t\right) l_t\right] \\[2mm] + M_t \mu_t \left[c_t + g_t + k_t - F\left(k_{t-1}, l_t, g_t\right) - (1-\delta) k_{t-1}\right] \\[2mm] + M_t \Gamma_t \left[V_t - u\left(c_t, l_t\right) - \beta \sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) \left\{m_{t+1} V_{t+1} + \theta m_{t+1} \ln m_{t+1}\right\}\right] \\[2mm] + \beta \sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) \Phi_{t+1}\left[M_{t+1} - m_{t+1} M_t\right] \\[2mm] + \beta M_t \sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) \omega_{t+1}\left[m_{t+1} - \dfrac{\exp\left(\frac{-V_{t+1}}{\theta}\right)}{\sum_{g_{t+1}} \pi(g_{t+1}|g^t) \exp\left(\frac{-V_{t+1}}{\theta}\right)}\right] \end{array} \right\}
$$
$$
- \xi u_c\left(c_0, l_0\right) \left[R_{b0} b_{-1} + \left\{1 + \left[1 - \Omega_0\right]\left[F_k\left(k_{-1}, l_0, g_0\right) - \delta\right]\right\} k_{-1}\right]
$$

The first-order necessary conditions for $t \geq 1$ are

$$
c_t : u_c\left(c_t, l_t\right) + \xi\left[u_{cc}\left(c_t, l_t\right) c_t + u_c\left(c_t, l_t\right) + u_{cl}\left(c_t, l_t\right) l_t\right] + \mu_t - \Gamma_t u_c\left(c_t, l_t\right) = 0 \tag{16}
$$

$$
l_t : u_l\left(c_t, l_t\right) + \xi\left[u_{cl}\left(c_t, l_t\right) c_t + u_{ll}\left(c_t, l_t\right) l_t + u_l\left(c_t, l_t\right)\right] - \mu_t F_l\left(k_{t-1}, l_t, g_t\right) - \Gamma_t u_l\left(c_t, l_t\right) = 0 \tag{17}
$$

$$
V_t : \Gamma_t - \Gamma_{t-1} + \left(\frac{1}{\theta}\right)\left[\omega_t - \sum_{g_t} \pi\left(g_t \mid g^{t-1}\right) m_t \omega_t\right] = 0 \tag{18}
$$

$$
k_t : \mu_t - \sum_{g_{t+1}} \beta \pi\left(g_{t+1} \mid g^t\right) m_{t+1} \mu_{t+1}\left[F_k\left(k_t, l_{t+1}, g_{t+1}\right) + 1 - \delta\right] = 0 \tag{19}
$$

$$
M_t : u\left(c_t, l_t\right) + \xi\left[u_c\left(c_t, l_t\right) c_t + u_l\left(c_t, l_t\right) l_t\right] - \sum_{g_{t+1}} \beta \pi\left(g_{t+1} \mid g^t\right) \Phi_{t+1} m_{t+1} + \Phi_t = 0 \tag{20}
$$

$$
m_{t+1} : -\Gamma_t\left[V_{t+1} + \theta\left(1 + \ln m_{t+1}\right)\right] - \Phi_{t+1} + \omega_{t+1} = 0 \tag{21}
$$

The $t = 0$ first order conditions, which are functions of the initial levels of capital and debt, are detailed in Appendix B.

There are two points worth noting about the set of optimality conditions. First, the first order conditions, and consequently the robustly optimal allocation, do not depend upon the level of the probability distortion, M_t. This result stems from the assumption that the government takes as its objective function

the consumers' subjective expected utility.[2]   Because the expectations of the two agents are aligned, the government does not attempt to use its policy tools to re-align the consumers' subjective expectation with the approximating probability model.   Rather, the government sets its taxes to induce the best path for the allocation and probability distortions, taking as given the current level of consumer beliefs.

Second, (18) indicates that the multiplier Γ_t is a martingale under the subjective expectation.   That is, $\tilde{E}_{t-1}\Gamma_t = \Gamma_{t-1}$.   A similar property is found in Svec (2011).   This martingale affects the persistence of the allocation.   In the limit as $\theta \to \infty$, the multiplier becomes constant over time and across states.

3.1.1   Ramsey Policies and Prices:

The solution to the Ramsey problem yields the equilibrium allocation and probability distortions.   The bond holdings in each state, then, are given by (15).   Given these values, this section describes the policies and prices that implement the solution.   That is, using the solutions that come from the Ramsey problem, the goal of this section is to determine the prices $\{w, r\}$, bond returns $\{R_b\}$, and taxes $\{\tau, \Omega\}$ that decentralize the equilibrium.   To accomplish this goal, I use the consumer's budget constraint and the first order conditions from the consumer's and the firm's problems.

The prices on capital and labor follow directly from the competitive firm's marginal product conditions. The labor tax rate can then be determined through the consumer's intra-temporal condition:

$$\tau_t = 1 + \frac{u_l\left(c_t, l_t\right)}{u_c\left(c_t, l\right) F_l\left(k_{t-1}, l_t, g_t\right)}$$

Thus, the intra-temporal wedge is uniquely pinned down by the allocation.

The two remaining variables to find are R_b and Ω.   The equations used to determine these values at time $t+1$ are

$$1 = \beta \sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) m_{t+1} \frac{u_c\left(c_{t+1}, l_{t+1}\right)}{u_c\left(c_t, l_t\right)} R_{b,t+1}$$

$$1 = \beta \sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) m_{t+1} \frac{u_c\left(c_{t+1}, l_{t+1}\right)}{u_c\left(c_t, l_t\right)} R_{k,t+1}$$

where

$$R_{k,t+1} = 1 + \left(1 - \Omega_{t+1}\right)\left(r_{t+1} - \delta\right)$$

and the $t+1$ consumer's budget constraint:

$$c_{t+1} + k_{t+1} + b_{t+1} - \left(1 - \tau_{t+1}\right) w_{t+1} l_{t+1} - R_{b,t+1} b_t - R_{k,t+1} k_t = 0 \tag{22}$$

As this set of equations makes clear, there are more unknowns than equations.   Consequently, this model cannot separately identify R_b and Ω.   To see this, suppose that there are N states of the world

---

[2] If, instead, the planner maximizes the expected utility of the consumers with respect to the approximating model, then the allocation would be a function of the distortion, M_t.

at time $t+1$. This means that there are $2N$ variables that must be pinned down and only $N+2$ equations. This indeterminacy is worsened by the fact that there is one additional linear dependency among the constraints. This can be seen by multiplying (22) by $\beta \sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) m_{t+1} u_c\left(c_{t+1}, l_{t+1}\right)$ and by summing the result over g_{t+1}. The outcome is a function only of the allocation and distortions and not $R_{b,t+1}$ or $R_{k,t+1}$. Thus, model uncertainty does not overturn the indeterminacy of the capital tax rates and debt returns, as found by Chari, Christiano, and Kehoe (1994).

An implication of this indeterminacy is that there are a number of different economic environments that would yield the same allocation. For example, if bond returns were assumed to be constant across states, then the government could still set the capital tax rates in such a way as to implement the Ramsey allocation. Conversely, if the government was unable to set state-contingent taxes on capital, then the allocation is still attainable by correctly varying the state-contingent returns on debt. More generally, any environment with an additional $N-1$ restrictions on capital taxes and debt returns would still lead to the Ramsey allocation being implemented.

The logic behind this result is as follows. Consumers choose to save in the capital and debt markets based on their subjective expectation of future returns in each of these markets. This means that the consumer's investment decision, for example, is a function of the weighted average of all capital returns at $t+1$. The same idea holds true for the debt market. Then, to encourage the correct level of savings, the government needs to focus only on the average returns to capital and government debt. This means that the fiscal authority has the flexibility to design the state-contingent nature of these returns in a number of ways, as long as the average returns on capital and debt are optimal and the consumer abides by her budget constraint.

Because of this indeterminacy, the state-contingent capital tax rates and bond returns cannot be separately identified. However, the theory pins down two policy variables related to these instruments. The first instrument is the ex-ante capital tax rate, defined as

$$
\Omega_t^e \equiv \frac{\sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) m_{t+1} \frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \Omega\left(g^{t+1}\right)\left[F_{k,t+1} - \delta\right]}{\sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) m_{t+1} \frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)}\left[F_{k,t+1} - \delta\right]} \tag{23}
$$

This ex-ante capital tax rate is the consumers' subjective expectation of the $t+1$ capital tax rate, weighted by the stochastic discount factor. Using (9), the numerator can be shown to equal

$$
\sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) m_{t+1} \frac{u_c\left(c_{t+1}, l_{t+1}\right)}{u_c\left(c_t, l_t\right)}\left[F_{k,t+1} + 1 - \delta\right] - \frac{1}{\beta}
$$

which is a function entirely of the allocation. Consequently, the ex-ante capital tax rate can be determined. This ex-ante value is different from the version in Chari, Christiano, and Kehoe (1994) in that the

expectation is taken with respect to the subjective probability model, rather than with the approximating model.

The second policy variable pinned down by the theory is labeled the private assets tax rate because it combines information from both the ex-post capital tax rate and the return on government debt. To derive this variable, suppose that the debt return in each state in period $t+1$ is the combination of a non-state-contingent return and a state-contingent tax rate:

$$R_{b,t+1} = 1 + \bar{r}_t \left[1 - \nu_{t+1} \right]$$

where the non-state-contingent rate of return, $\bar{r}_t$, must satisfy

$$\sum_{g_{t+1}} \pi \left(g_{t+1} \mid g_t \right) m_{t+1} \frac{u_c \left(c_{t+1}, l_{t+1} \right)}{u_c \left(c_t, l_t \right)} R_{b,t+1} = \sum_{g_{t+1}} \pi \left(g_{t+1} \mid g_t \right) m_{t+1} \frac{u_c \left(c_{t+1}, l_{t+1} \right)}{u_c \left(c_t, l_t \right)} \left[1 + \bar{r}_t \right]$$

This constraint implies that

$$\sum_{g_{t+1}} \pi \left(g_{t+1} \mid g_t \right) m_{t+1} \frac{u_c \left(c_{t+1}, l_{t+1} \right)}{u_c \left(c_t, l_t \right)} \nu_{t+1} = 0$$

With this decomposition, the non-state-contingent return on debt can be determined through (8). From the government's budget constraint, the total tax revenues from capital and debt in a particular state g_{t+1}

$$\Omega_{t+1} \left[r_{t+1} - \delta \right] k_t + \nu_{t+1} \bar{r}_t b_t$$

are equal to

$$g_{t+1} - \tau_{t+1} w_{t+1} l_{t+1} - b_{t+1} + \left(1 + \bar{r}_t \right) b_t$$

Finally, in order to turn this value into a rate and ease comparisons to the ex-ante capital tax rate, divide by the total return across capital and bonds in each state. Then, the private assets tax rate is

$$\eta_{t+1} = \frac{\Omega_{t+1} \left[r_{t+1} - \delta \right] k_t + \nu_{t+1} \bar{r}_t b_t}{\left[r_{t+1} - \delta \right] k_t + \bar{r}_t b_t}$$

Overall, this fiscal policy model with capital pins down the wage, the rental rate of capital, and three tax variables: a labor tax, the ex-ante capital tax, and a private assets tax. In order to determine the specific characteristics of these prices and policies, I will construct the recursive version of the planner's optimization problem and numerically solve it using value function iteration. But, before I follow this procedure, there is one policy result that can be analytically derived by focusing attention on a specific, and simple, class of functions describing the consumers' preferences. I highlight this implication in the following section.

3.1.2 Ex-Ante Capital Tax Rate under Preference Restrictions:

A powerful finding of Chari, Christiano, and Kehoe (1994) is that, within a specific class of utility functions, the ex-ante capital tax rate is exactly equal to zero. However, one might fear that this policy conclusion hinges upon the assumption that consumers have rational expectations. In this section, I re-examine whether this theoretical implication still survives when consumers face model uncertainty.

For this section, assume that the utility function of the consumers is quasi-linear, where

$$u(c, l) = c + v(l)$$

Plugging this functional form into the consumer's first order condition with respect to capital for $t > 0$, the equation becomes

$$1 = \beta \sum_{g_{t+1}} \pi(g_{t+1} \mid g_t) m_{t+1} \{1 + (1 - \Omega_{t+1})(F_k(k_t, l_{t+1}, g_{t+1}) - \delta)\}$$

The planner's first order condition with respect to the same variable is

$$1 = \beta \sum_{g_{t+1}} \pi(g_{t+1} \mid g^t) m_{t+1} \frac{\mu_{t+1}}{\mu_t} [F_k(k_t, l_{t+1}, g_{t+1}) + 1 - \delta]$$

where μ_t is the Lagrange multiplier on the resource constraint in period t. Combining these two equations with (16), the numerator of the ex-ante capital tax rate is equal to

$$\beta \sum_{g_{t+1}} \pi(g_{t+1} \mid g_t) m_{t+1} \left[\frac{\Gamma_t - \Gamma_{t+1}}{\Gamma_t - 1 - \xi} \right] [F_k(k_t, l_{t+1}, g_{t+1}) + 1 - \delta] \tag{24}$$

Proposition 2 $\forall t > 1$, *if* $cov_t \{(\Gamma_t - \Gamma_{t+1}), (F_{k,t+1} + 1 - \delta)\} = 0$ *under the consumer's subjective expectation, then* $\Omega_t^e = 0$. $\Omega_t^e \neq 0$ *otherwise.*

Proof. *To see this, first note that* Γ_t *is a martingale under the consumer's subjective expectation, where* $\widetilde{E}_t \Gamma_{t+1} = \Gamma_t$. *Then, a property of covariance suggests that the numerator is equal to*

$$\frac{\beta}{\Gamma_t - 1 - \xi} cov_t \{(\Gamma_t - \Gamma_{t+1}), (F_{k,t+1} + 1 - \delta)\}$$

It follows from (23) that $\Omega_t^e = 0$ *only when* $cov_t \{(\Gamma_t - \Gamma_{t+1}), (F_{k,t+1} + 1 - \delta)\} = 0$ *and* $\Omega_t^e \neq 0$ *when this condition does not hold.* ∎

This proposition provides a simple test to determine whether the value of the ex-ante capital income tax rate is equal to 0 for a given value of θ. In the limit as $\theta \to \infty$, the Lagrange multiplier Γ_t is constant across time $\Gamma_t = \Gamma, \forall t$. This implies that the covariance is equal to zero and hence the ex-ante capital tax rate is also equal to 0. This is the case examined by Chari, Christiano, and Kehoe (1994). Outside of this limit, though, the covariance is no longer equal to zero, meaning that the ex-ante capital tax rate is also non-zero.

Intuitively, this result stems from the fact that the planner must consider how its choice of capital taxes affects the consumers' incentive to save as well as their endogenous beliefs. This second desire can be seen through the first term in the covariance: $\Gamma_t - \Gamma_{t+1}$. This random variable tracks the shadow value of the consumers' welfare across states, which, in turn, reflects the consumers' probability distortion across those same states. In balancing these two incentives, the government allows the shadow value of the consumers' welfare, Γ, to fluctuate.

Another perspective confirms the logic underlying the proposition. It can be shown that if

$$\frac{\mu_{t+1}}{\mu_t} = \frac{u_c\left(c_{t+1}, l_{t+1}\right)}{u_c\left(c_t, l_t\right)} \tag{25}$$

then $\Omega_t^e = 0, \forall t \geq 1$. That is, if the planner places the same value on resources over time as the consumer, then the ex-ante capital tax rate is equal to 0. This condition is satisfied in a rational expectations model. However, when consumers face model uncertainty, the planner values resources differently than the consumers. This is because the planner, when considering whether to allocate more consumption to the consumers in one state, takes into account not just the consumers' marginal utility gain from that action, but also the effect that action has on the consumers' probability distortion. It is this additional marginal value that breaks the equality in (25). Thus, there is no theoretical presumption that the ex-ante capital tax rate is equal to 0, even under quasi-linear preferences.[3]

3.2   Recursive Formulation of Ramsey Problem:

This section describes the recursive formulation of the planner's problem. Government spending is now assumed to follow a Markov process. The natural state vector is a function of both capital and government spending. However, because of the forward-looking constraint on the movement of the consumers' subjective welfare, V_t, this problem is not time-consistent. As detailed by Marcet and Marimon (1998), the addition of a co-state variable allows this constraint to be written recursively. The co-state variable, Γ, keeps track of the past promises made by the planner about the consumers' subjective welfare.

The time 0 values of the capital stock, debt, and probability distortion imply that the period 0 problem of the government is unlike the problem it faces in all other periods. To account for this difference, the recursive formulation has to be separated into two. The first Bellman equation presented below applies to the planner's problem in any period $t > 0$, while the second one applies only to $t = 0$. When calculating

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[3]Although I have written the proof assuming a quasi-linear form of consumer preferences, a similar argument can be made for a utility function of the following form:

$$u\left(c, l\right) = \frac{c^{1-\sigma}}{1 - \sigma} + v\left(l\right)$$

The only difference is that (24) would contain the ratio $\frac{u_{c,t+1}}{u_{c,t}}$ in the expectation, which, in turn, would modify the covariance term in the proof.

the path of the economy over time, the values of the endogenous variables coming from the $t = 0$ problem will be used as inputs into the $t > 0$ problem.

The planner's value function, $H\left(\cdot;\xi\right)$ satisfies the following Bellman equation:

$$
H\left(k_{-},\Gamma_{-},g_{-};\xi\right) = \min_{\Gamma_g}\ \max_{c_g,l_g,V_g,k_g,m_g}\ \sum_{g}\pi\left(g\mid g_{-}\right)
\left\{
\begin{array}{c}
m_g u\left(c_g,l_g\right) + \xi m_g\left[u_c\left(c_g,l_g\right)c_g + u_l\left(c_g,l_g\right)l_g\right] \\[4pt]
+ m_g\mu_g\left[c_g + g + k_g - F\left(k_{-},l_g,g\right) - \left(1-\delta\right)k_{-}\right] \\[4pt]
- \Gamma_{-}\left[m_g V_g + \theta m_g \ln m_g\right] + m_g\Gamma_g\left[V_g - u\left(c_g,l_g\right)\right] \\[4pt]
+ \omega_g\left[m_g - \dfrac{\exp\left(\frac{-V_g}{\theta}\right)}{\sum_{g}\pi(g|g_{-})\exp\left(\frac{-V_g}{\theta}\right)}\right] + \beta m_g H\left(k_g,\Gamma_g,g;\xi\right)
\end{array}
\right\}
$$

There are many points worth noting here. First, this Bellman equation is written from an ex-ante perspective. This formulation is necessary because of the presence of the incremental probability distortion. As noted above, this distortion is a function of the characteristics across all states within the same time period. In order to capture this, the Bellman equation must be expressed before the realization of uncertainty. Thus, the subscript g denotes the state-contingent value of each random variable.

Second, the solution to this problem is indexed by the multiplier ξ. For a given ξ, the first order conditions and additional constraints imply an optimal allocation. This allocation is used to construct the implementability constraint, including the time 0 values of the allocation. If the implementability constraint is satisfied with equality at that ξ, then the resulting allocation satisfies all constraints and yields the highest subjective welfare for the consumers. If the implementability constraint is slack, then the algorithm will keep searching over ξ until it finds the solution.

The time 0 recursive problem of the planner is

$$
H_0 = \min_{\Gamma_0}\ \max_{c_0,l_0,V_0,k_0}
\left\{
\begin{array}{c}
u\left(c_0,l_0\right) + \xi\left[u_c\left(c_0,l_0\right)c_0 + u_l\left(c_0,l_0\right)l_0\right] - \xi u_c\left(c_0,l_0\right)\left[R_{b,0}b_{-1} + R_{k,0}k_{-1}\right] \\[4pt]
+ \mu_0\left[c_0 + g_0 + k_0 - F\left(k_{-1},l_0,g_0\right) - \left(1-\delta\right)k_{-1}\right] \\[4pt]
+ \Gamma_0\left[V_0 - u\left(c_0,l_0\right)\right] + \beta H\left(k_0,\Gamma_0,g_0\right)
\end{array}
\right\}
$$

where $R_{k,0} = 1 + \left(1-\Omega_0\right)\left(F_k\left(k_{-1},l_0,g_0\right) - \delta\right)$. The first order conditions for both of these recursive problems are detailed in appendix C. There, they are verified to be equivalent to those derived in the sequential formulation of the Ramsey problem.

4 Numerical Findings:

To numerically solve this model, I apply a value function iteration algorithm to the $t > 0$ Bellman equation of the government. The state space is assumed to be bounded and rectangular. For an initial ξ, the algorithm iterates until the value function has converged. Using this value function, I solve the $t = 0$

recursive problem. The solution to this Bellman equation yields the initial values of the allocation, as well as the values of the state variables that are inputted into the $t > 0$ Bellman equation. With these values, I then construct the implementability constraint, assuming that the infinite time constraint can be approximated by T periods. The program loops over ξ until the implementability constraint is satisfied with equality. At this value, the solution fully solves the planner's problem.

Before getting to the numerical solutions, it is helpful to understand the logic of the rational expectations model in order to better distinguish the implications of model uncertainty. When the consumers have rational expectations, the benevolent planner must use its policy to mitigate the welfare costs associated one type of distortion: the assumed linearity of the taxes on capital and labor. These linear taxes distort the savings and consumption / labor margins, respectively. In response to this distortion, Chari, Christiano, and Kehoe (1994) show that the government optimally sets the ex-ante capital tax rate to 0. This choice leaves the savings margin undistorted. An implication of this result is that, on average, the government does not use its tax on capital income to finance government spending. Rather, the government uses the labor tax to fulfill this goal. Although this leads to a large distortion in the consumers' consumption / labor decision, the government reduces the welfare cost of this distortion by implementing a relatively smooth labor tax rate across states. Since government spending is volatile while labor taxes are relatively smooth, the government finances the shock to its spending through large fluctuations in capital taxes and bond returns. Thus, the government lowers the costs of the distortion by setting a fairly smooth labor tax and a volatile private assets tax, while maintaining an expected capital tax rate equal to zero.

In addition to the linearity of the tax rates, model uncertainty adds an additional distortion to the analysis. The consumers, in their uncertainty about the shock process, distort their subjective probability model away from the approximating model. The resulting pessimism not only alters the consumers' decisions, but also reduces their subjective expected utility. This second point is due to the fact that consumers place a smaller subjective probability on the high welfare state occurring and a greater subjective probability on the low welfare state occurring. The government then must use its fiscal policy to reduce the welfare costs associated with both the linear taxes and consumer uncertainty.

To help elucidate the resulting optimal policy, I will graph the numerical solutions that come from the value function iteration algorithm described above. In calculating these solutions, I assume that the consumers' preferences are described by

$$u\left(c,l\right) = \left(1 - \gamma\right)\log c + \gamma\log\left(1 - l\right)$$

and the production function is of the form

$$F\left(k,l\right) = k^{\alpha}l^{1-\alpha}$$

These assumptions follow Chari, Christiano, and Kehoe (1994). Government spending follows the process

$$g_t = \bar{g} + \rho \left(g_{t-1} - \bar{g} \right) + \epsilon$$

where ϵ is drawn from an approximation to a normal distribution, $\epsilon \tilde{} N \left(0, \sigma^2 \right)$. Depending on the value of ρ, this process could resemble an iid shock to spending or an AR(1) process. For my numerical calculations, I allow the shock to take three possible values. Then, to simplify the exposition, I choose to plot only the two outside values of ϵ. This decision is not costly, since the middle value of ϵ corresponds with the middle value of V, which leads to essentially no probability distortion.

At this point, I make the following strong assumption: I assume that the approximating model happens to be correct. This assumption implies that the government's confidence is well-placed, while the consumers' uncertainty is harmful. In fact, with this assumption, the consumers would have been better off if they were more confident in the approximating probability model.

The parameters assumed in the numerical simulations are

Utility and Technology Parameters		Government Spending Parameters		Initial Values	
γ	0.25	$\bar{g}$	0.2	$R_{b,0} b_{-1}$	0
β	0.98	ρ	0	k_{-1}	2.5
α	0.34	σ	0.12	Ω_0	1
δ	0.08			g_0	$\bar{g}$

In the following figures, I graph the equilibrium solutions for many different levels of consumer uncertainty at a particular point in time. A large θ implies that the consumers are relatively confident in the approximating model. A small θ means that the consumers are uncertain about a large range of models surrounding the approximating model. This latter case will lead the government to use its fiscal policy more aggressively in order to mitigate the welfare costs of uncertainty.

During the transition period, model uncertainty does not qualitatively change the optimal policies implemented by the planner. The government sets a negative tax on labor income at $t = 0$, and the ex-post tax on capital income is fixed exogenously. The ex-ante capital tax rate in period $t = 0$, Ω_0^e, is very large and on the order of 1000%. As consumers become increasingly uncertain, the government raises Ω_0^e slightly. Just as in Chari, Christiano, and Kehoe (1994), this expected tax is large because it is a function of the initial capital stock, an inelastic variable.

Next, I turn to the policies implemented by the government after the initial period. As indicated earlier, there are three policy variables pinned down by this model: the ex-ante capital tax, the labor tax, and the private assets tax. Given that the most striking result in Chari, Christiano, and Kehoe (1994) is that the expected capital income tax rate equals zero for a specific class of utility functions, I will present this tax rate first.

From the analytical section, we know that the government no longer sets the ex-ante capital tax rate equal to zero after the initial period when faced with consumer uncertainty. The magnitude and the direction of this deviation from zero is shown in Figure 1. As this figure indicates, the government on average chooses to subsidize capital income when facing consumer uncertainty. This action will, ceteris paribus, encourage the consumers to increase their investment in capital and public debt. Notably, the size of this optimal subsidy is relatively modest. With the chosen set of parameter values, the government's maximum subsidy is approximately 2%. Evidently, consumer uncertainty does not provide a significant justification for allowing the ex-ante capital income tax rate to diverge far from zero.

In addition to the expected capital tax rate, two other policies were derived in the analytical section: a labor income tax and a tax on private assets. These policies are plotted in Figure 2. There are two points worth noting about the government's choice of labor income taxes. First, the average labor tax is large and positive across all levels of θ. This fact implies that the government should obtain the majority of its revenue from taxing labor, regardless of the level of consumer uncertainty. Second, consumer uncertainty leads the government to reduce the degree to which the labor tax acts as a shock absorber for government spending. To see this, note that the covariance between the labor tax rate and government spending falls as θ falls, becoming negative for large enough values of consumer uncertainty. This negative covariance reduces the degree to which the government can rely on labor tax revenues to finance the deviation of public spending from its mean.

The opposite conclusions are drawn from the graph depicting the private assets tax. This tax on capital and debt is centered around zero, implying that the government collects little revenue, on average, from the private assets tax. However, consumer uncertainty leads the government to increase the covariance between the private assets tax and government spending. This policy suggests that the government should rely more heavily on debt returns and capital taxes to finance the deviation of spending from its mean when consumers face model uncertainty.

A more direct method to examine the shock absorbing properties of the labor and private assets tax is to answer the following question: assuming $g = g_h$, how much of the increase in government spending from its mean is financed using the labor and private assets tax? Given the information in the previous figure, one would expect that the percentage of the shock financed through the labor tax (private assets tax) falls (rises) with consumer uncertainty. The results, graphed in Figure 3, are consistent with this belief.

The previous three figures have plotted the ex-ante capital income tax rate, the labor income tax rate, and the private assets tax rate. Each of these policy instruments were pinned down in the analytical section above. The model, though, was unable to pin down the fourth policy instrument: the ex-post capital income tax rate. To derive this variable, I must make the additional assumption that the return

on public debt is non-state contingent. Making this additional assumption, I have plotted the state-contingent ex-post capital tax rate in Figure 4. The profile of the ex-post capital tax rate mirrors the characteristics of the private assets tax. The ex-post capital income tax is not used to finance the average size of public spending, but rather is used to absorb the deviation of spending from its mean. This shock absorbing property becomes increasingly relevant as consumer uncertainty rises.

In summary, there are three main policy implications of consumer uncertainty: first, the government should subsidize capital income, though at a modest rate; second, the covariance of the labor income tax with government spending should fall as the level of consumer uncertainty rises; and third, the covariance of the private assets tax with government spending should rise with the level of consumer uncertainty. Although these policy responses describe how the government optimally responds to consumer uncertainty, they do not indicate why the government has chosen these responses. The next few paragraphs describe the rationale behind the government's decisions and provide supporting evidence for this interpretation.

Consumer uncertainty, as seen through the lens of this altruistic government, is costly because it induces the consumers to behave in a pessimistic fashion. This pessimism makes the consumers reluctant to save in capital or public debt because they worry that the true probability model is one that yields a low rate of return on these investments. The resulting reduction in savings is detrimental for two reasons. First, the fall in savings reduces the economy's capital stock. This, in turn, decreases key macroeconomic variables like output, wages, and consumption. Second, if the consumers save less, it becomes more difficult for the government to finance its spending shock. Because of both of these costs, the government must use its policy instruments to increase how much consumers invest in capital and public debt.

One key fiscal instrument that influences the consumers' savings decisions is the private assets tax. If the government wants to raise the level of consumer savings in both capital and public debt, the government should reduce the expected value of the private assets tax. This reduction would increase the after-tax return on these assets, thus inducing consumers to invest more in capital and debt. As Figure 5 shows, this is exactly how the government responds to consumer uncertainty. Specifically, as consumer uncertainty rises, the government reduces the average value of the private assets tax across states.[4]

This policy decision, though, comes with a cost: by choosing to subsidize savings, the government must obtain more tax revenue from another source in order to finance its spending. In this case, the government offsets the subsidies by increasing the average labor income tax rate, as seen in Figure 6. Comparing Figures 5 and 6, we can see that, as consumer uncertainty rises, the government both increases the subsidy on private assets and the average labor tax rate. This increase in the average labor tax is

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[4]I use the average value of the private assets tax rather than the expected value because the former isolates the impact of consumer uncertainty on the tax rates. That is, the average value is useful because it doesn't combine information on both the tax rates and the stochastic discount factor. Later in this discussion, I describe how the movement in the stochastic discount factor is consistent with the movement in the tax rates.

beneficial in that it allows the government to subsidize savings. But, it is costly in that it further distorts the consumption - leisure tradeoff away from the first-best. As the average labor tax grows, the distortion becomes increasingly harmful to the consumers' welfare. Interestingly, it is this rising welfare cost that leads the government to find a second method of encouraging consumers to save without resorting to raising the subsidy on capital income.

This second method – one that induces the consumers to save more without requiring a higher average labor income tax – involves using policy to manipulate the consumers' subjective probability model. In particular, the government reduces the covariance between the labor income tax and government spending, as shown in Figure 2. This policy reduces the size of the welfare gap $V(g_L) - V(g_H)$, which can be seen in Figure 7. This change in the consumers' welfare profile is important because it leads the consumers to decrease (increase) the subjective weight they place on the high (low) spending state, relative to the weights the consumers would otherwise place on the two states. That is, because the high spending state is no longer associated with a large reduction in welfare, the consumers need not fear this state as much, and as such, $m(g_H)$ falls. The converse holds true for the low spending state. In Figure 8, I provide evidence of these subjective probability movements by plotting the consumers' subjective probability distortion. In that figure, I compare the consumers' probability distortion under the optimal policy and under a counterfactual scenario. In the counterfactual scenario, I assume that, for all levels of θ, the government maintains the same gap in the consumers' subjective welfare as is optimal when $\theta \to \infty$. This comparison is necessary to show that the state-contingent probability distortions are smaller than they would be under an alternative (non-optimal) policy.

Upon noting that the high (low) government spending state is associated with a large and positive (large and negative) ex-post capital income tax, we see the benefit of the government's policy: by inducing the consumers to lower (raise) their subjective weight on g_H (g_L), the government effectively lowers the expected value of the capital income tax without modifying the underlying state-contingent capital income taxes. With this knowledge, we can now re-analyze Figure 1. That figure shows that the government lowers the ex-ante capital income tax rate as consumer uncertainty increases. In fact, the fall in the ex-ante capital income tax depicted in Figure 1 is the product of two policies. The first policy is that the government directly lowers the average level of the private assets tax, and by extension, the capital income tax rate. The second policy is that the government modifies the state-contingent profile of labor taxes to induce the consumers into believing that the state associated with the low capital income tax is relatively more likely than the state associated with the high caiptal income tax.

In conclusion, the numerical results suggest that the government responds to consumer uncertainty by encouraging the consumers to increase their savings in capital and public debt. The government accomplishes this by directly lowering the tax on private assets and by manipulating the consumers'

expectations.

Until now, I have described how consumer uncertainty affects optimal fiscal policy and the consumers' allocation at a particular point in time. Now, I change my focus in order to highlight the impact of uncertainty on the time series properties of policy and the allocation. Specifically, Figure 8 plots how key economic variables respond to a one-period increase in government spending. To help clarify the impact of uncertainty on the impulse response functions, I have drawn two lines for each graph in Figure 8. The solid line displays the baseline scenario in which the consumers face no uncertainty, while the dotted line displays the numerical solutions when consumers face a substantial amount of model uncertainty. By comparing these two lines, we can determine how uncertainty affects the equilibrium.

The top left graph plots the shock to government spending. With this one-period increase in spending, the government lowers the labor income tax relative to the baseline model. This fall leads to an increase in the consumers' labor supply, which increases the economy's output. In addition, the fall in the labor tax rate lowers both the wage and the labor tax revenues. We can also see that the labor income tax rate remains persistently lower than the tax rate in the baseline model. This leads to persistently higher values of labor supply and output.

The welfare implications of this fiscal policy model depend on the assumed spending shock process. In this paper, I have assumed that the true government spending process happens to be the approximating model, implying that the government's confidence is well-placed. Model uncertainty, then, is harmful to the consumers, since they are guarding against alternative probability models that turn out to be incorrect. This loss in welfare can be seen in Figure 9, which plots the consumers' subjective welfare as a function of their model uncertainty. The consumers' subjective welfare falls as θ falls, and then only moves upward when policy is used so aggressively that the consumers weight the high government spending state less highly than the low spending state.

5 Conclusion:

This paper examines how consumer uncertainty affects the optimal policies implemented by a fiscal authority in a model with capital. Unlike in a rational expectations framework, consumers lack confidence about the equilibrium probability model. Wanting to be robust against this uncertainty, they apply a max-min operator to their decision problems. That is, the consumers choose the allocation that maximizes their expected utility, where the expectation is taken with respect to their subjective probability model. Additionally, it is assumed that the government is fully confident that the approximating model correctly characterizes the stochastic environment.

This confidence dichotomy implies that an altuistic government could have one of a number of possible

objective functions. In this analysis, it is assumed that the government maximizes the consumers' expected utility under their own subjective probability model. This assumption aligns the expectations of the two agents. Given these preferences, the government seeks to use its fiscal policy to mitigate the welfare costs associated with both the assumed linearity in the tax rates and consumer uncertainty. It is shown analytically that, under one condition, the government no longer implements a zero ex-ante capital tax rate. This is because the government takes into account how the consumers' allocation affects their probability distortion.

The numerical implementation of this model shows that the optimal ex-ante capital tax rate is a modest subsidy on the order of 2%. To finance this subsidy, the government raises the average value of the labor income tax. Further, the numerical results suggest that, relative to the full-confidence benchmark model, the government increases the covariance of the private assets tax and lowers the covariance of the labor income tax with respect to government spending. In doing so, the government manipulates the consumers' subjective expectation, making them believe that the subsidy is relatively large. These policies encourage the consumers to invest more in both capital and public debt, which helps counter the consumers' pessimistic response to uncertainty.

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7 Appendix A:

In this appendix, I characterize and solve the Ramsey problem of a government that maximizes the consumers' expected utility under the approximating model. In order to simplify the formulation, let

$$W\left(c_t, l_t, M_t, \xi\right) \equiv u\left(c_t, l_t\right) + \xi M_t\left[u_c\left(c_t, l_t\right) c_t + u_l\left(c_t, l_t\right) l_t\right]$$

This term combines the consumers' period utility function with the implementability constraint. With this definition, the sequential problem is

$$\mathcal{L} = \sum_{t=0}^{\infty}\sum_{g^t}\beta^t\pi\left(g^t\right)\left\{ \begin{array}{c} W\left(c_t, l_t, M_t, \xi\right) \\[6pt] +\mu_t\left[c_t + g_t + k_t - F\left(k_{t-1}, l_t, g_t\right) - \left(1-\delta\right) k_{t-1}\right] \\[6pt] +\Gamma_t\left[V_t - u\left(c_t, l_t\right) - \beta\sum_{g_{t+1}}\pi\left(g_{t+1}\mid g^t\right)\left\{m_{t+1}V_{t+1} + \theta m_{t+1}\ln m_{t+1}\right\}\right] \\[6pt] +\beta\sum_{g_{t+1}}\pi\left(g_{t+1}\mid g^t\right)\Phi_{t+1}\left[M_{t+1} - m_{t+1}M_t\right] \\[6pt] +\beta\sum_{g_{t+1}}\pi\left(g_{t+1}\mid g^t\right)\omega_{t+1}\left[m_{t+1} - \dfrac{\exp\left(\frac{-V_{t+1}}{\theta}\right)}{\sum_{g_{t+1}}\pi(g_{t+1}\mid g^t)\exp\left(\frac{-V_{t+1}}{\theta}\right)}\right] \end{array}\right\}$$

$$-\xi u_c\left(c_0, l_0\right)\left[R_{b0}b_{-1} + \left\{1 + \left[1-\Omega_0\right]\left[F_k\left(k_{-1}, l_0, g_0\right) - \delta\right]\right\} k_{-1}\right]$$

The first-order necessary conditions for $t \geq 1$ are

$$c_t : W_c\left(c_t, l_t, M_t, \xi\right) + \mu_t - \Gamma_t u_c\left(c_t, l_t\right) = 0$$

$$l_t : W_l\left(c_t, l_t, M_t, \xi\right) - \mu_t F_l\left(k_{t-1}, l_t, g_t\right) - \Gamma_t u_l\left(c_t, l_t\right) = 0$$

$$V_t : \Gamma_t - m_t\Gamma_{t-1} + \left(\frac{1}{\theta}\right) m_t\left[\omega_t - \sum_{g_t}\pi\left(g_t\mid g^{t-1}\right) m_t\omega_t\right] = 0$$

$$k_t : \mu_t - \sum_{g_{t+1}}\beta\pi\left(g_{t+1}\mid g^t\right)\mu_{t+1}\left[F_k\left(k_t, l_{t+1}, g_{t+1}\right) + 1 - \delta\right] = 0$$

$$M_t : \Phi_t + W_M\left(c_t, l_t, M_t, \xi\right) - \sum_{g_{t+1}}\beta\pi\left(g_{t+1}\mid g^t\right)\Phi_{t+1}m_{t+1} = 0$$

$$m_{t+1} : -\Gamma_t\left[V_{t+1} + \theta\left(1 + \ln m_{t+1}\right)\right] - \Phi_{t+1}M_t + \omega_{t+1} = 0$$

The first order condition with respect to V_t indicates that the multiplier Γ_t is a martingale under the approximating model: $E_{t-1}\Gamma_t = \Gamma_{t-1}$. This martingale affects the persistence of the allocation. In the limit as $\theta \to \infty$, this martingale condenses to a constant.

The first order conditions also suggest that the allocation is a function of the consumers' probability distortion, M_t. This result is due to the misalignment of the government's expectation and the consumers. That is, the government incorporates the approximating model into its objective function, even though the consumers behave as if this approximating model is not correct. This misalignment implies that the

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planner must keep track of how the chosen allocation affects the probability distribution of the consumers. This is accomplished through the term M_t. An implication of this is that M_t is a state variable in the recursive problem of the government.

The recursive problem of the government is

$$H\left(k_-, M_-, \Gamma_-, g_-; \xi\right) = \min_{\Gamma_g} \max_{c_g, l_g, M_g, V_g, k_g, m_g} \sum_g \pi\left(g \mid g_-\right) \left\{ \begin{array}{c} W\left(c_g, l_g, M_g, \xi\right) + \mu_g\left[c_g + g + k_g - F\left(k_-, l_g, g\right) - (1-\delta) k_-\right] \\ -\Gamma_-\left[m_g V_g + \theta m_g \ln m_g\right] + \Gamma_g\left[V_g - u\left(c_g, l_g\right)\right] \\ +\Phi_g\left[M_g - m_g M_-\right] + \omega_g\left[m_g - \dfrac{\exp\left(\frac{-V_g}{\theta}\right)}{\sum_g \pi(g|g_-) \exp\left(\frac{-V_g}{\theta}\right)}\right] \\ +\beta H\left(k_g, M_g, \Gamma_g, g; \xi\right) \end{array} \right.$$

where $W\left(c_g, l_g, M_g, \xi\right) \equiv u\left(c_g, l_g\right) + \xi M_g\left[u_c\left(c_g, l_g\right) c_g + u_l\left(c_g, l_g\right) l_g\right]$

8 Appendix B:

The $t = 0$ first order conditions for a government that maximizes the consumers' expected utility under the distorted probability model are

$$c_0 \quad : \quad 0 = u_c(c_0, l_0) + \xi [u_{cc}(c_0, l_0) c_0 + u_c(c_0, l_0) + u_{cl}(c_0, l_0) l_t]_0 + \mu_0 - \Gamma_0 u_c(c_0, l_0)$$

$$-\xi u_{cc}(c_0, l_0) [R_{b0} b_{-1} + R_{k0} k_{-1}]$$

$$l_0 \quad : \quad 0 = u_l(c_0, l_0) + \xi [u_{cl}(c_0, l_0) c_0 + u_{ll}(c_0, l_0) l_0 + u_l(c_0, l_0)] - \mu_0 F_l(k_{-1}, l_0, g_0) - \Gamma_0 u_l(c_0, l_0)$$

$$-\xi u_{cl}(c_0, l_0) [R_{b0} b_{-1} + R_{k0} k_{-1}] - \xi u_c(c_0, l_0) (1 - \Omega_0) F_{lk}(k_{-1}, l_0, g_0)$$

$$V_t : 0 = \Gamma_0$$

$$k_0 : 0 = \mu_0 - \sum_{g_1} \beta \pi(g_1 \mid g_0) m_1 \mu_1 [F_k(k_0, l_1, g_1) + 1 - \delta]$$

9 Appendix C:

The first order conditions from the recursive formulation of the planner's problem are

$$c_g : 0 = u_c\left(c_g, l_g\right) + \xi\left[u_{cc}\left(c_g, l_g\right) c_g + u_c\left(c_g, l_g\right) + u_{cl}\left(c_g, l_g\right) l_g\right] + \mu_g - \Gamma_g u_c\left(c_g, l_g\right)$$

$$l_g : 0 = u_l\left(c_g, l_g\right) + \xi\left[u_{cl}\left(c_g, l_g\right) c_g + u_l\left(c_g, l_g\right) + u_{ll}\left(c_g, l_g\right) l_g\right] - \mu_g F_l\left(k_-, l_g, g\right) - \Gamma_g u_l\left(c_g, l_g\right)$$

$$k_g : 0 = \mu_g + \beta H_k\left(k_g, \Gamma_g, g; \xi\right)$$

$$V_g : 0 = -\Gamma_- + \Gamma_g + \left(\frac{1}{\theta}\right)\left[\varpi_g - \sum_g \pi\left(g \mid g_-\right) m_g \varpi_g\right]$$

$$m_g : \quad 0 = u\left(c_g, l_g\right) + \xi\left[u_c\left(c_g, l_g\right) c_g + u_l\left(c_g, l_g\right) l_g\right] - \Gamma_-\left[V_g + \theta\left(1 + \ln m_g\right)\right]$$
$$+ \Gamma_g\left[V_g - u\left(c_g, l_g\right)\right] + \varpi_g + \beta H\left(k_g, \Gamma_g, g; \xi\right)$$

$$\Gamma_g : 0 = V_g - u\left(c_g, l_g\right) + \beta H_\Gamma\left(k_g, \Gamma_g, g; \xi\right)$$

The envelope conditions are

$$H_k\left(k_-, \Gamma_-, g_-; \xi\right) = -\sum_g \pi\left(g \mid g_-\right) \mu_g m_g\left[F_k\left(k_-, l_g, g\right) + 1 - \delta\right]$$

$$H_\Gamma\left(k_-, \Gamma_-, g_-; \xi\right) = -\sum_g \pi\left(g \mid g_-\right)\left[m_g V_g + \theta m_g \ln m_g\right]$$
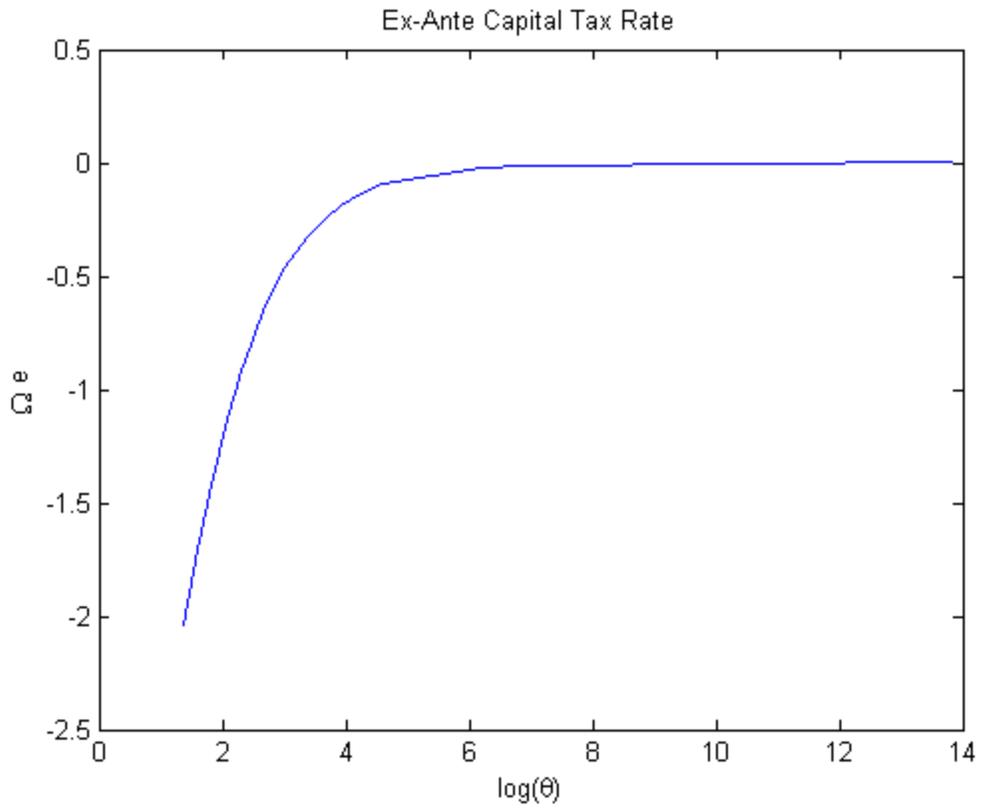
Figure 1: Ex-ante capital tax rate, t≥1, across different levels of consumer uncertainty
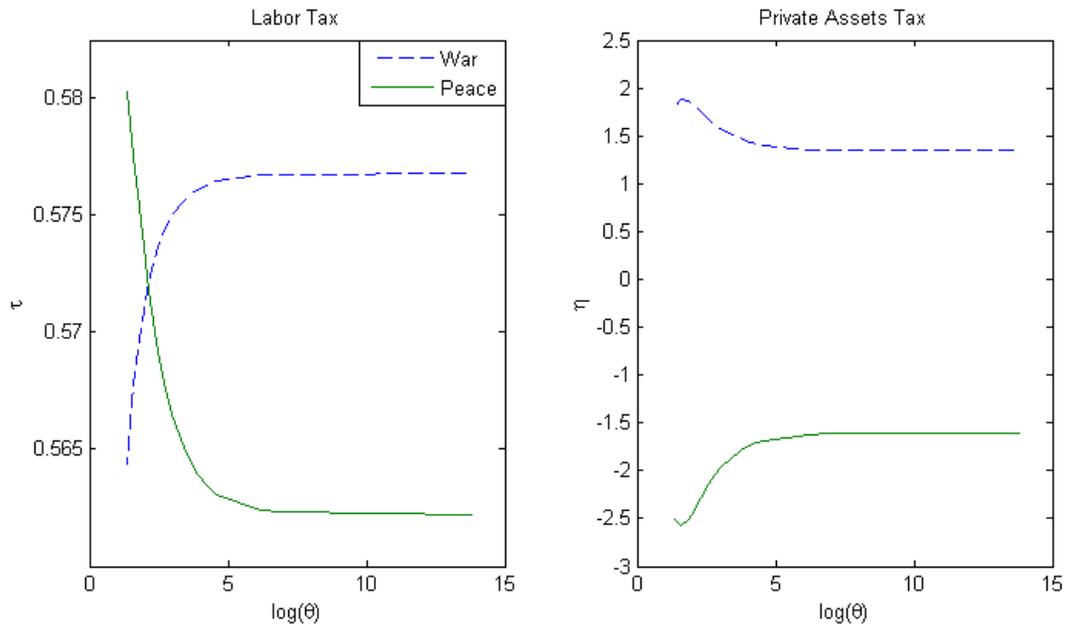
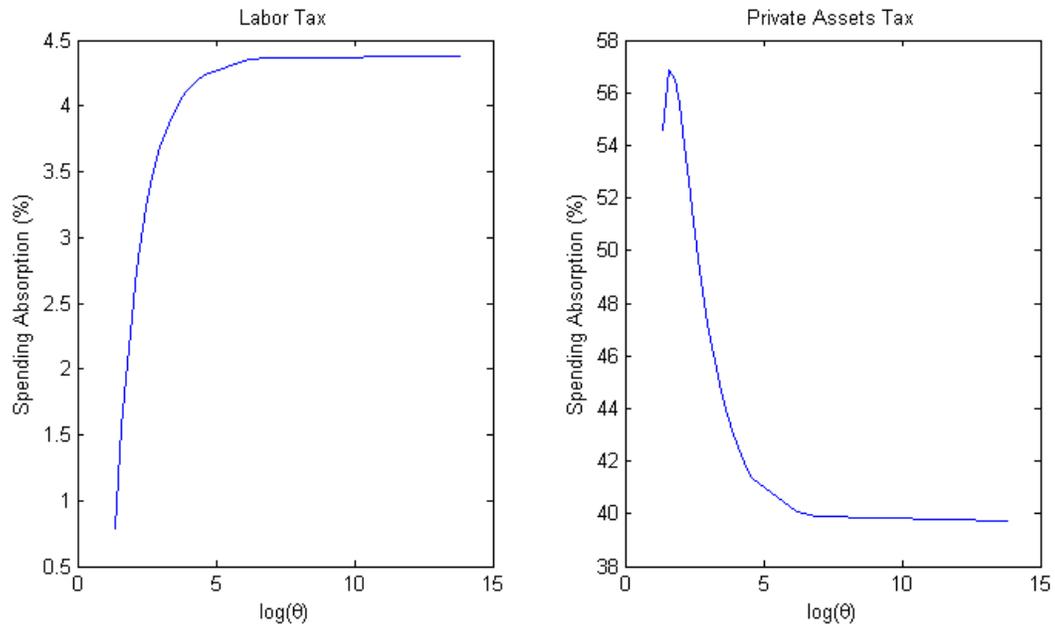Figure 2: Labor income and private assets taxes across different levels of consumer uncertainty

Figure 3: Percentage of an increase in government spending financed through labor and private assets taxes across different levels of consumer uncertainty
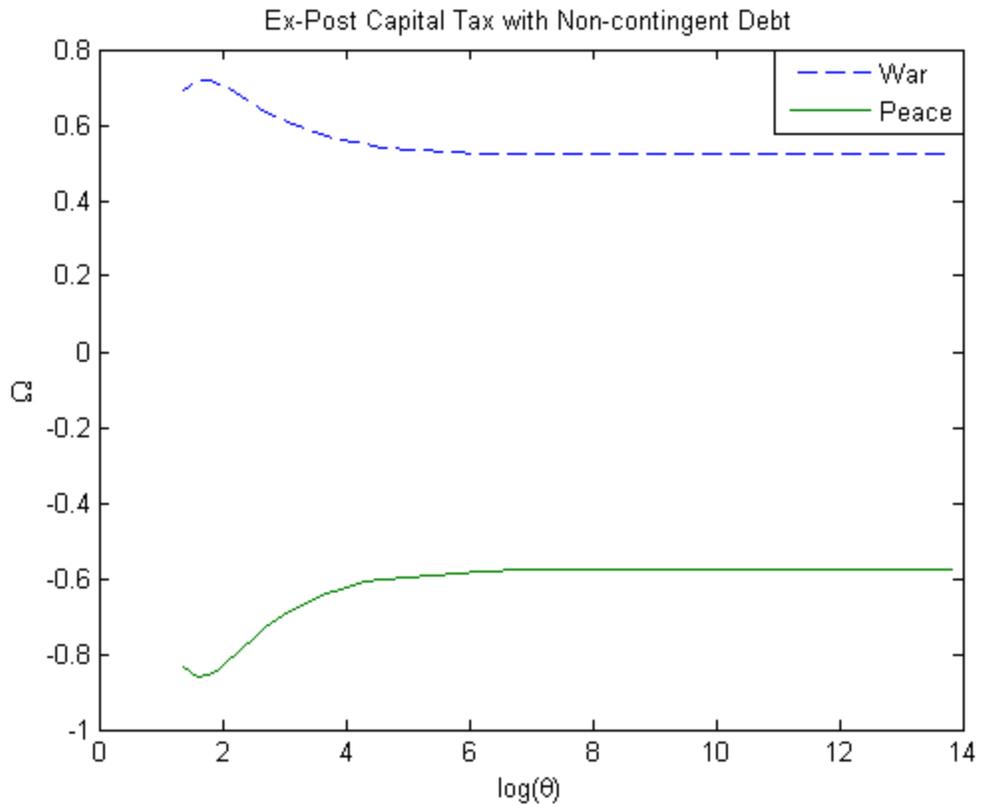
Figure 4: Ex-post capital tax assuming non-state-contingent debt returns across different levels of consumer uncertainty
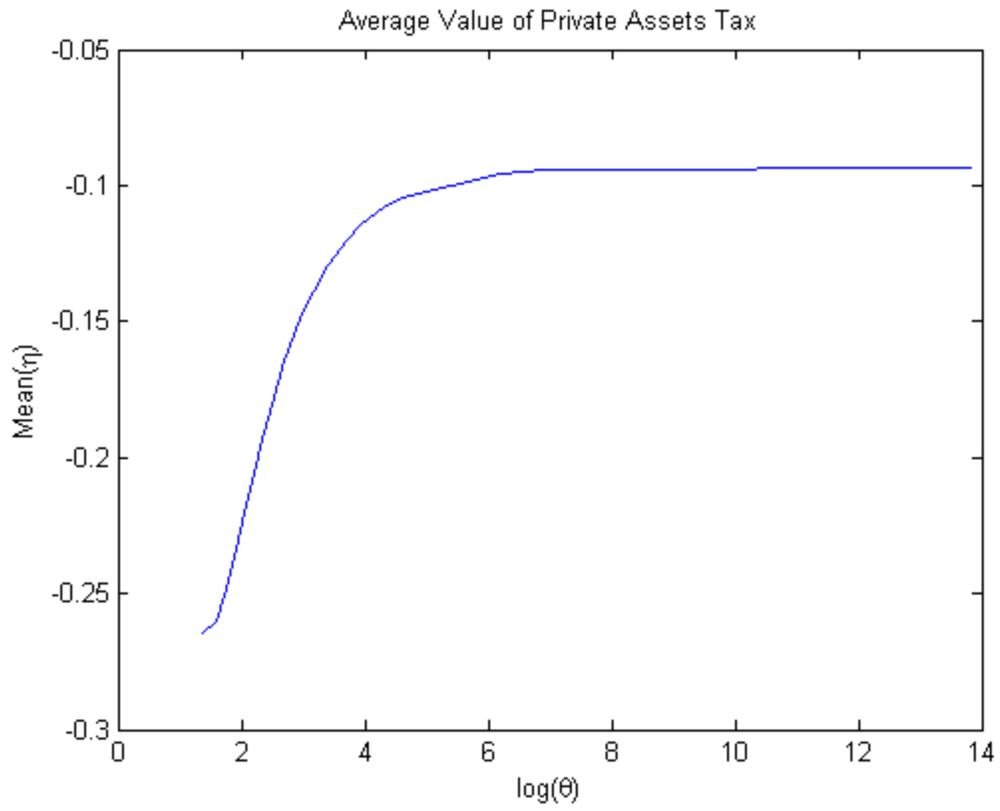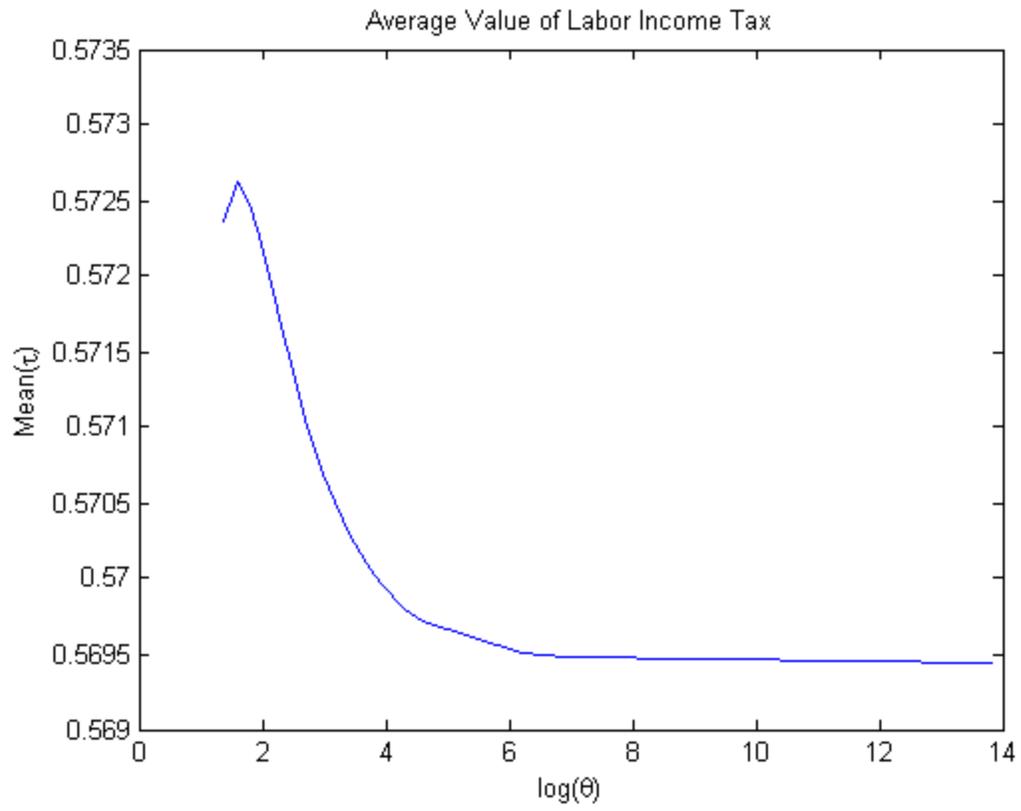
Figure 5: The average value of the private assets tax across different levels of consumer uncertainty

Figure 6: The average value of the labor income tax across different levels of consumer uncertainty
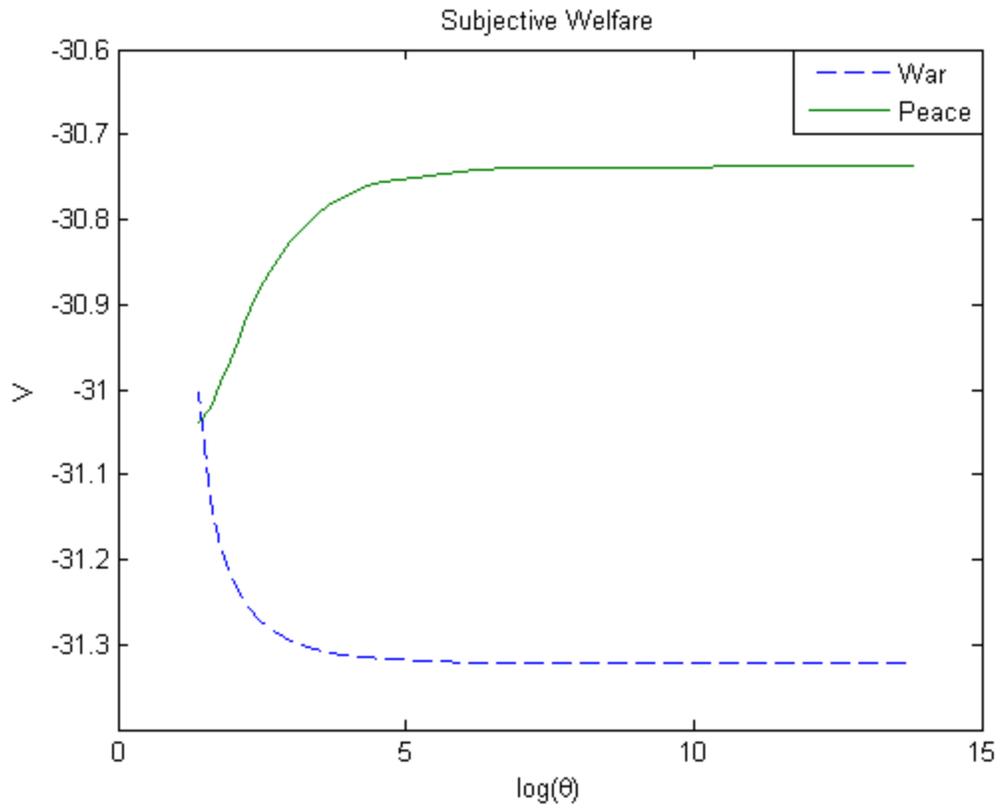
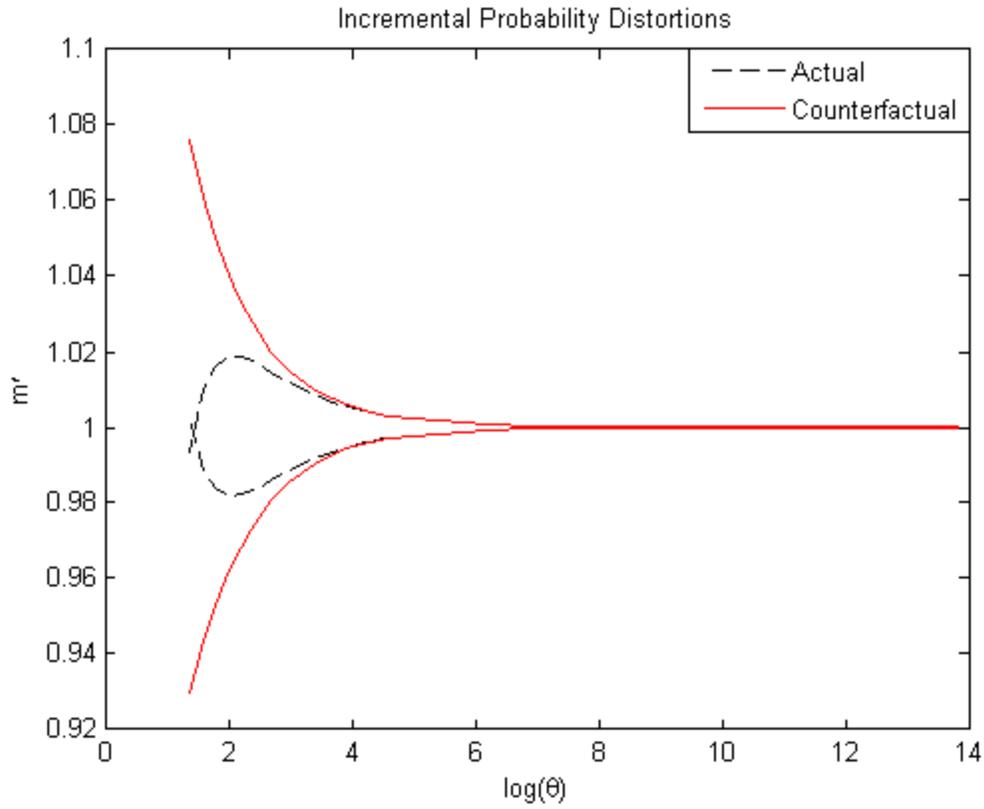Figure 7: The consumers' subjective welfare profile across different levels of consumer uncertainty

Figure 8: The consumers' actual subjective probability weights, m', and the subjective probability weights under the counterfactual scenario in which the profile of V remains the same as under rational expectations
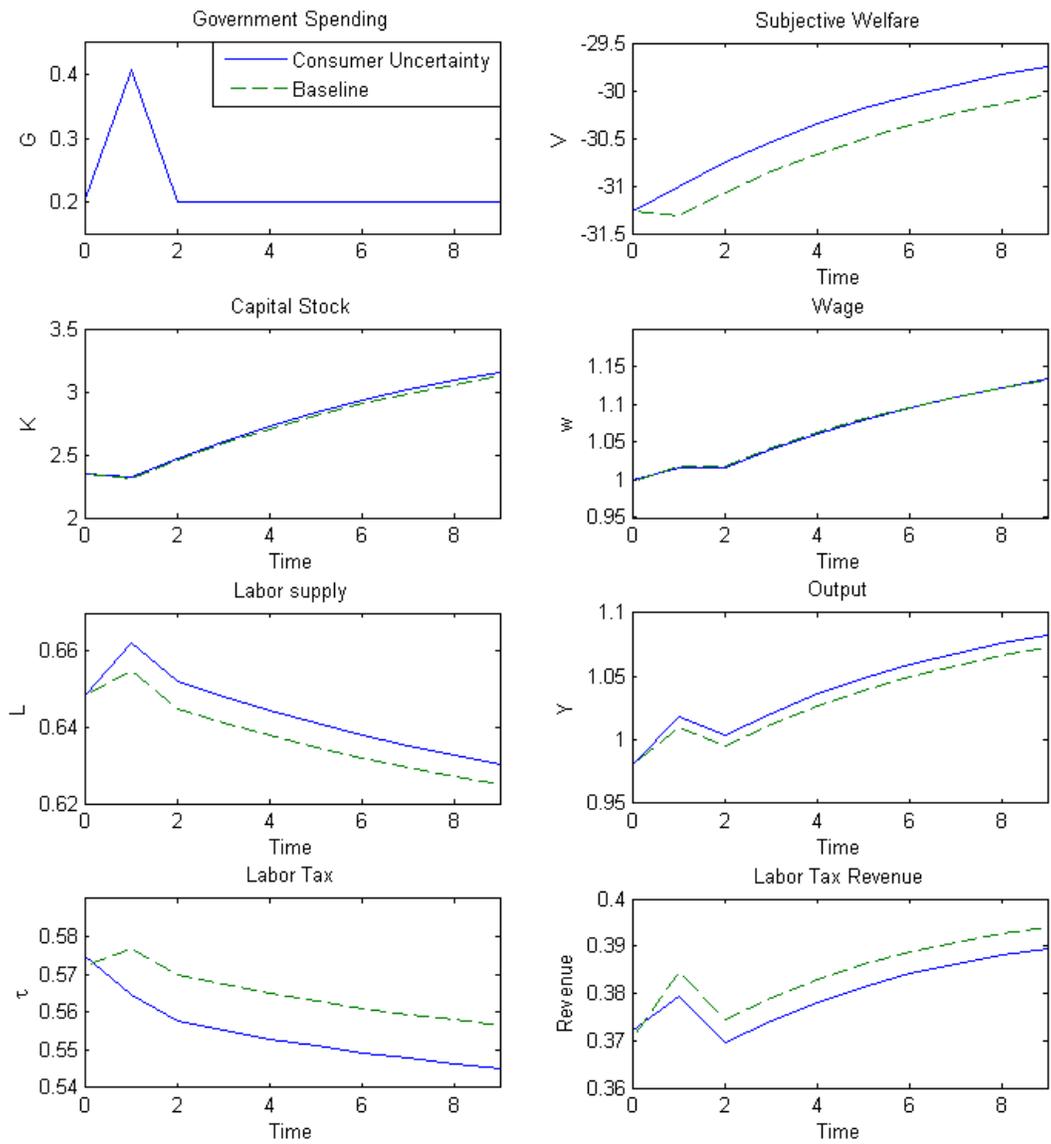
Figure 9: The impulse response functions for a one period increase in government spending
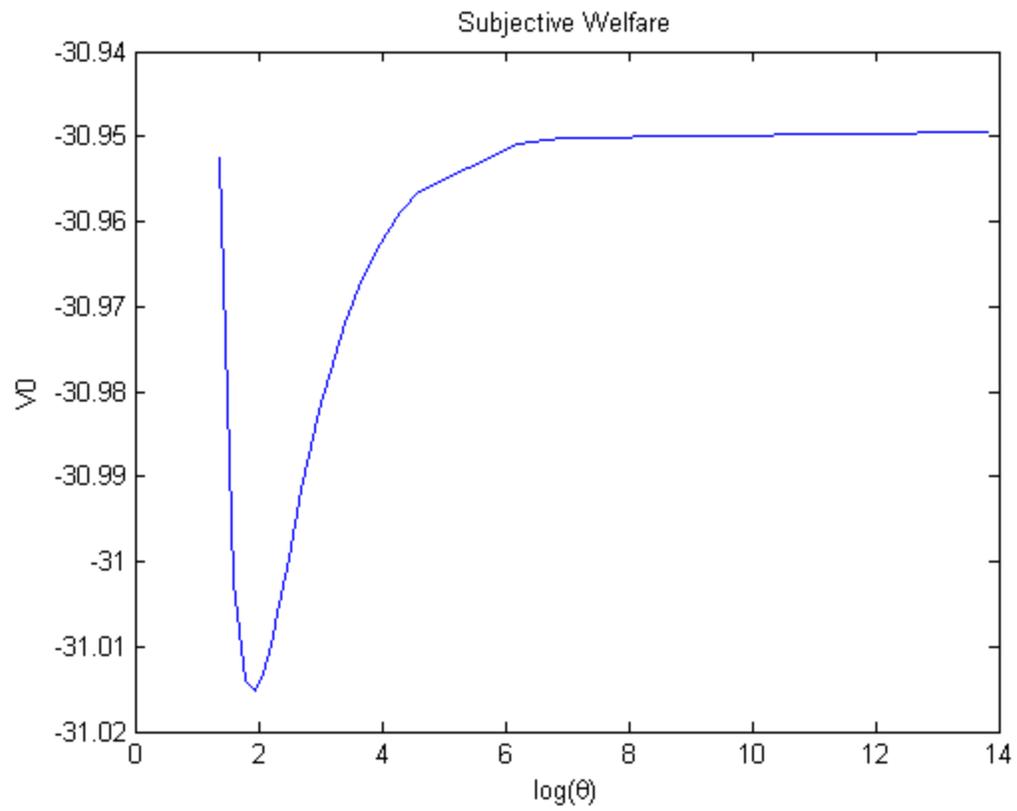
Figure 10: The consumers' subjective welfare at t=0 across different levels of consumer
uncertainty