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The Ultimate Tradeoff for Colleges: Academic Quality or Consumption Amenities?

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Economics Honors Thesis
12/16/2017
Introduction:

As a college student, I overhear my fellow classmates engage in frequent conversations about the recent rise in tuition expenses and the avenues in which their money is spent within college operations. These comments inspired my research in this paper. More specifically, my focus was to delve into the finances of American institutions of higher education and to observe where money is actually being spent and which areas of the college money is being dispersed. Furthermore, I wanted to use this information in order to examine whether or not students are actually stimulating their own tuition growth through their costly demands on colleges and the luxury services colleges are now offering. In particular, this paper analyzes the current-day trade off for American colleges: spending on consumption amenities as opposed to spending on academic quality. From my research and conclusions, it seems as though institutions of higher education are prioritizing development in different areas, apart from academic quality and instructional development, in order to suite the needs and desires of millennial college prospects.

Background:

For more than a quarter-century now, American students seeking higher education have confronted a significant problem: a seemingly steady rise in tuition expenses. According to Donna Desrochers of the American Institutes for Research, tuitions (inflation-adjusted) at four-year public institutions in the United States have increased nearly 160 percent since 1990 (Desrochers, 2014). Similarly, Nobel Prize-winning economist Joseph Stiglitz asserts in his book, The Price of Inequality: How Today’s Divided Society Endangers Our Future, that the inflation-adjusted tuition in some states has increased by 104 percent in public two-year schools
and by 72 percent in public four-year schools between 2007-2008 and 2012-2013 (Stiglitz, 2013). When compared to the CPI for all urban consumers of education, school fees, and childcare, I find that there is an increase in the CPI from 100 in 1990 to 399.76 in 2015. This constitutes a net increase of CPI of 299.76 over the twenty-five year time span. Found on the Federal Reserve Bank Database, this data is summarized in a graph in the attached appendix. As evidenced by these numbers, the escalation in the price of higher education has sparked much public concern and raises legitimate public policy questions about both the causes and ramifications of this trend. Calls for “free college” and outrage over mounting student debt levels both played a large role in the recent 2016 Presidential campaign, highlighting the fact that mounting prices have sparked national discussions. Since parents, students, and families are the ones who must find ways to pay the escalating tuition bills, many have begun to question the value of pursuing a college degree, especially in traditional humanities or other “soft” disciplines.

Due to this questioning of the value of a college degree, colleges have become apprehensive about their long-term solvency. Small private institutions, especially those that are less well-known and heavily tuition-dependent, worry about their survival, particularly if steps are taken to make “tuition-free” college a reality in some form or fashion. At large, America’s colleges and universities are constantly faced with economic tradeoffs. In the short-term, they must pay their bills, meet payroll, and balance their budgets. In the long-term, however, they must maintain viability and remain attractive to students, in order to ensure that a demand for their main product, an education, always exists. However, colleges do not merely provide an educational product to their students, but also a memorable experience. In a way, the pool of consumption amenities offered at a particular institution relates directly to the type of experience
a student will have at the institution they choose. These two products offered by colleges, an education and an experience, are the roots of the ultimate tradeoff colleges are currently facing.

Therefore, colleges must plan for the future, be innovative, anticipate future needs, and ensure their survival by planning for continued growth. They must remain financially viable, while also maintaining their academic quality and reputations in the eyes of prospective students. For some institutions, raising, and not simply maintaining, their reputations present new challenges. The college “ratings game,” as trumpeted by national ranking systems like U.S. News and World Report and Princeton Review, further exacerbates problems. The need to preserve and “grow” one’s reputation can be one factor responsible for rising tuition costs.

According to Michael Luca and Jonathon Smith in their paper, *Salience in Quality Disclosure: Evidence from the U.S. News College Rankings*, a one-rank improvement on indexes like U.S. News and World Report leads to a one-percentage-point increase in applications received by the particular college in a given year (Luca, 2013). Under these circumstances, it becomes evident that the challenges of remaining financially solvent, while simultaneously attracting students, are sometimes in conflict. An often popular method to remaining financially solvent is to cut costs, while remaining attractive to students requires continual investment. These conflicting factors will be discussed throughout this paper, since evidence suggests that colleges could be cutting costs from their academic sectors so as to invest in student demands and services.

Additionally, the trend of using sparkling new campus facilities as a means to attract students has further exacerbated the tuition problem. The erection of new buildings has sparked a “multiplying effect” across institutions in America. If a college increases its spending so as to increase its overall quality, competing colleges feel the need to continue the trend to uphold relative prestige (Renehan, 2015). This multiplying effect advances the notion that institutions
are not merely competing for students on the basis of tuition, but also on the supply of consumption amenities offered at the particular school. This trend can be seen through the results of Barbara Tobolowsky and John Lowery’s paper, *Selling College: A Longitudinal Study of American College Football Bowl Game Public Service Announcements*. The researchers specifically studied institutionally-created commercials and noted the types of attributes that the colleges used to “brand” themselves (Tobolowsky, 2014). Along with rankings and other indicators of institutional excellence, colleges showcased their traditional architectural elements, beautiful campuses, and impressive student achievements in these PSAs (Tobolowsky, 2014). This finding is of particular importance to my study because it indicates that colleges do, in fact, value a diverse branding of their institution by placing an importance on not only academic rankings, but also on aesthetic topographies.

The above information provides considerable background on the central focus of my paper. With tuition revenue, endowments, and capital funding constrained and limited funds available to institutions of higher education, colleges cannot choose to fund all the projects that they deem as valuable. In general, colleges have three options in their expenditures. They could allocate money to academic means, they could spend money on consumption amenities, or they could raise their tuitions so as to maximize the provisions of both. While the latter demonstrates to be very unpopular to the general public, colleges have been slowly raising their tuitions year by year. Yet, colleges are aware that increasing tuition costs contribute to the crowd-out of students within the market for higher education. Therefore, colleges must make a trade off with the funds they have available and choose among competing and costly projects, funding some and sacrificing others. As an example, colleges could choose to direct money towards building state-of-the-art residence halls, dining venues, food courts, recreational facilities – things that are
collectively called consumption amenities – or towards improving the academic quality of the institution.

Both of these options, either spending towards academics or amenities, have certain benefits and costs associated with them. If a college decides to spend more on consumption amenities, this would heavily attract the students who have personalized preferences for elements which enhance the college experience, such as luxury and entertainment items. By contrast, if a college decides to spend more on academic quality, this may attract students who primarily value the benefits of a college education, rather than the extraneous enhancements of the college experience. The chosen spending policy of a particular college has the benefit of attracting the students who most closely share the preferences of the college, while, at the same time, has the cost of potentially alienating students who share dissimilar preferences.

While colleges do, in fact, have the option of spending their tuition dollars in ways which would enhance the academic quality and the education that students receive, current research suggests that colleges are choosing to contribute to amenities over academic quality. In other words, the experience wins out in the trade-off between a college education or an experience, between academics or amenities. The evidence shows that there are many ways in which colleges are allocating money towards amenities, both directly and indirectly, and deducting finances away from academic purposes. Colleges are choosing to act in this manner in order to suite the preferences of prospective college students. In other words, colleges are deducing that students are extremely sensitive to the vast array of amenities offered at their particular school, rather than to the quality of their academic professors and provisions.
In fact, not only does the literature indicate that students are responsive to tuition prices, but the body of literature also reveals that students are sensitive to a college’s supply of amenities as well. American businessman Peter Schwartz, for instance, argues that students respond to prices and are heavily price-sensitive (Schwartz, 2011). However, differing interpretations highlight the possibility that students are sensitive to factors apart from institutional prices. In fact, campus administrators have reported that current prospects actually “expect” high-end amenities and consider them to be integral aspects of the college experience (Gordian, 2016). State-of-the-art residence halls, dining venues, food courts, recreational facilities, and added luxuries have become prioritized in the eyes of college applicants. In his book, College Tuition: Four Decades of Financial Deception, Robert Iosue offers his insight on this issue when he states, “Students are always seeking additional services and amenities and colleges are often too quick to accommodate them with little regard to the financial consequences” (Iosue, 2014). The amenities race between institutions and the expectations on schools to provide extraneous luxuries to their students are developments which Iosue has labeled the “Edifice Complex.” This issue of the Edifice Complex plays an increasingly important role in struggle of colleges to balance their finances and to attract prospective students.

As the Gordian Company states in its 2016 annual report, “The State of Facilities in Higher Education,” “Campuses are in a seemingly unsustainable race to both “catch up” and “keep up” as they compete to attract the best students” (Gordian, 2016). This “amenities race” between institutions of higher education has profound implications for other aspects of colleges, including employment and campus development. More specifically, the report comments on the facility age and use on college campuses and highlights the recent trend in building projects. While older buildings are in desperate need of maintenance and renovation, investment in these
buildings has been postponed in favor of the construction of new facilities. Most important, however, is the fact that 50 percent of recent campus growth is attributed to buildings used for nonacademic purposes (Gordian, 2016). The report asserts, “This is part of a 100-year trend to make campuses more residential and entice prospective students with better housing, dining, and other support services” (Gordian, 2016). In addition to taking on new debt to finance these construction projects, colleges are increasingly relying on cutting costs in other areas of the college to support the need for campus expansions.

For example, colleges may be incentivized to hire more part-time faculty, as opposed to full-time faculty, in light of increasing student demands for luxury amenities and for campus developments. This relationship between the hiring of part-time faculty and consumption amenities is important to consider for two reasons. First, colleges are increasingly turning towards part-time faculty and second, there is research to suggest that this move is not coincidental or innocuous. According to the American Institutes for Research, staffing patterns in higher education have experienced significant changes over the past two decades. The replacement of full-time, tenured faculty by part-time adjuncts or graduate assistants has profound implications for both institutional spending and tuition prices. Colleges have become more heavily-reliant on part-time professors so as to cut costs on salaries and benefits (Desrochers, 2015). Coupled with the surge in adjunct hires, institutions have also augmented their pools of non-teaching staff and administrators, who cater to student demand.

After considering all of the connected factors mentioned above, I build a theoretical model based on the intuition in the Calvert-Wittman model, which explores the policy decisions of two candidates in a presidential election. My model, in turn, explores the incentives that colleges face when choosing between funding consumption amenities and academic quality. In
this model, there are two colleges who compete with each other on the basis of number of students who attend and of ideal policies for spending on education. Colleges can either be more motivated by students or by policy, or can allocate equal weights to both factors. Additionally, I give one college a valence advantage of a historic reputation for academic quality over the other. I show that if the valence characteristics of the two colleges are approximately equal, the colleges will choose different spending policies than if one college holds a valence advantage as compared to the other. As a college’s valence increases, this causes the school to be more encouraged to choose its ideal policy, rather than choosing the policy which more closely resembles that of its opposing school. I show that the size of the valence advantage matters, since a smaller valence advantage will incentivize the advantaged school to mimic the policy of its opponent. In the case of a large valence, the disadvantaged school will actually benefit more from an extreme policy than a moderate one and should choose a policy on the polar end of the spectrum.

Additionally, I consider a spectrum of students who have their own individualized preferences for a college’s spending policy and who value the exogenous reputation of the college they attend. I explicitly consider the case of the breakeven student, or the student who will be indifferent between one college and the other. I show how the valence advantage of a school can influence the decision-making of a prospective student. In this environment, I look for the Nash equilibrium set of spending policies for the colleges and examine how this equilibrium depends on the location of the ideal spending policies of the colleges as well as the historic reputation of each college.

As a brief preview of my results, I find the Nash equilibriums for the cases mentioned above. I consider various weights that colleges may place on both students and policy. In the
case when colleges only care about the number of students who attend their school and do not value their spending policy at all, they will act in the way which will maximize the number of student enrollees. Both colleges will place themselves at the median spending policy of 50%. On the contrary, if colleges only value their ideal spending policies, rather than the number of students who attend their schools, then they should always choose their ideal spending policies to maximize their utilities.

In the real world, however, colleges will place their own respective weights on students and on policy. My results are convincing and compelling in the sense that they highlight the current trends we observe in the research. If colleges place a high or even a medium level of priority on the number of students who attend their school, I find that colleges will be forced to sacrifice their ideal spending mix in favor of a more moderate and equalized mix of amenities to consumption spending. If colleges do not moderate their spending policies under these circumstances, they will be crushed by the market preference for equalized spending. Only schools with very large valence advantages will be able to choose their ideal spending policies without implementing any damage to their values or their preferences. For instance, a school which is ideologically extreme in terms of its spending policy on academics should only implement this policy if its valence advantage is large enough to withstand the damage it would feel from a loss in students (or in the unlikely case the school did not care about students at all). When valence is large, students will attend regardless of the extremity of the college’s policy.
**Literature Review:**

These issues in higher education have been studied and discussed in previous literature. More specifically, other economists have researched the spending patterns of colleges and their tendencies of allocating financial resources to consumption amenities, while diverting resources away from academic purposes. As discussed below, the vision of college as a “country club,” in the eyes of prospective students could be connected to recent trends in adjunct instruction and the studied effects of this instruction on academic spending.

It is important to highlight that my study is not the first to examine tradeoffs faced by colleges. More specifically, economists Brian Jacob, Brian McCall, and Kevin Stange have furthered Robert Iosue’s idea of a problematic “Edifice Complex” among American institutions in their paper, *College as Country Club: Do Colleges Cater to Students’ Preferences for Consumption?*. My research and theoretical model is focused on extending the earlier findings and ideas of these authors. Even though their thesis question is tested through both a theoretical and an empirical approach, their theoretical model provides significant insights as to the channel in which a school will choose to allocate its resources. In their model, they assume that colleges provide only two attributes: academic quality and consumption amenities. From this assumption, it is concluded that the optimal ratio for spending between academic quality and consumption amenities for a particular institution is dependent upon four factors: the enrollment elasticity with respect to consumption, the enrollment elasticity with respect to academic quality, and the costs per student for both academic quality and amenities (Jacob, 2013). This finding appears to be significant during an age of high tuition expenses and steadily-decreasing enrollment numbers. If enrollment targets are not met, the existing student population will expect and demand the same level of amenities and services. To increase enrollment in subsequent years, institutions may
need to boost its spending on amenities and perhaps offer new ones in the search for new “customers.” For example, if a particular institution attracts a large amount of amenity-sensitive students, the total enrollment and tuition revenue the institution receives will be heavily swayed by the large demand shifts stemming from additional amenities. The authors then tested their theory through an analysis of student willingness to pay by school and an assessment of how the school should respond to such willingness.

The empirical results of their paper offer important insights as to the actual priorities of prospective students throughout their college search and to the compromising tradeoffs institutions are currently facing. Colleges compete for students on multiple dimensions: tuition price, academic quality, consumption amenities, and proximity from home. However, students respond differently to these attributes depending on their specific preferences. Even though student preferences appear to be extremely heterogeneous, the authors concluded that preferences for consumption amenities are considerably universal across all prospective students (Jacob, 2013). This conclusion is congruent with my assertions above; Colleges realize that students are extremely sensitive to the vast array of amenities offered at the particular school and are altering their spending policies on the basis of this knowledge.

By contrast, an overarching preference for academic quality can only be seen amongst an exclusive portion of the population: high-income, high-ability students (Jacob, 2013). The conclusion here is that many more students make choices based on amenities available to them, rather than on any pre-conceived notion of academic quality. It becomes evident through these results that colleges face another unfortunate tradeoff. Institutions can choose to spend money on academic purposes, but doing so may not yield a student body of the size they need to sustain
themselves. They are, instead, caught in a continuous cycle of competition and of debt through consumption amenities.

From these results, the question arises as to whether students are prioritizing their instant satisfaction over the long-term, educational benefits of an institution. One possible explanation for this dynamic is that students are uninformed about the academic quality of a particular school or are unable to make accurate assessments about what exactly constitutes “academic quality” in a college or a university. Schwartz emphasizes this point, arguing that students encounter difficulty both in choosing an institution based on quality of academic programs and in assessing the future benefit of their investment in higher education (Schwartz, 2011). Likewise, Jacob, McCall, and Stange argue that the majority of students give precedence to their “quality of life” at the present moment, rather than to the unseen benefits of a quality education which will not surface until after graduation.

Another question that must be asked is this: exactly what constitutes “academic quality?” In their paper, Jacob, McCall, and Stange assess the academic quality of an institution based on instructional spending and academic support. A decline in instructional spending can be represented through the hiring of more part-time professors, in an effort to decrease costs for the purpose of allocating funds to different areas. This choice on the part of colleges to decrease academic spending through the channel of part-time professors has considerate implications on student behavior in the future.

Eric Bettinger and Bridget Terry Long explore this question of adjunct quality and expenditures in their paper, *Do College Instructors Matter? The Effects of Adjuncts and Graduate Assistants on Students’ Interests and Success*. Assuming higher-quality faculty are
more expensive than part-time adjuncts, the authors empirically tested the effects on students after having been taught by an adjunct in a particular course. Their results indicate that part-time faculty appear to have a statistically-significant, negative effect on the number of subsequent courses students choose to take in the subject (Bettinger, 2004). This finding leads readers to believe that part-time adjuncts differ in instructional quality in comparison to full-time professors and reduce the overall academic quality of an institution when more adjuncts are hired.

Another problem that must be considered is the explosion of non-teaching staff on college campuses. In order to satisfy students’ desires for luxuries, amenities, and services, institutions have excessively expanded their pools of administrative positions, which have directly increased salary expenditures for schools. Professional positions, which offer non-instructional services to students, now account for between 20-25 percent of on-campus jobs (Desrochers, 2015). The financing of these salaries, coupled with a steady decrease in enrollments and tuition revenues, has placed colleges in the position to cut costs from academic staffing. A potential method to cut costs from the academic sector is to hire part-time professors, who can be denied health benefits and can be paid considerably less than full-time faculty. Additionally, the public belief that increases in tuition are padding the pockets of full-time professors is truly a misconception. Between the years of 2002-2010, faculty salaries at public bachelor’s universities actually remained constant, while the tuition at these institutions rose five percent (Desrochers, 2015). From these facts, it can be further deduced that American colleges have been altering their spending policies. Increased spending on amenities and decreased expenditures on academics have been the recent and reoccurring trends.

While Kevin Stange, Brian McCall, and Brian Jacob’s ideas in their paper, *College as Country Club: Do Colleges Cater to Students’ Preferences for Consumption?*, form the
underlying motivation for my analysis, the foundation of my theoretical model borrows heavily from a variant of the standard Downsian Model. In the standard Downsian model, politicians value getting elected and enacting a policy that is close as possible to their own preferred one. Given these preferences, it can be shown that the politicians choose to announce a policy platform that matches the median voter’s preferred policy, a result called the median voter theorem. That is, despite wanting to choose a policy that conforms to their own preferences, the politicians are forced to choose the median voter’s preferred policy in order to have any hope of getting elected.

The variation to this model that is most important for my paper is described and analyzed in Groseclose’s 2001 paper, *A Model of Candidate Location When One Candidate Has a Valence Advantage*. This model extends the Downsian model by assuming that political candidates could have a “valence advantage.” These valence advantages are described as non-policy factors which can attract a voter to a particular candidate, such as campaign funds, name recognition, charisma, or intelligence (Groseclose, 2001). This addition to the Calvert-Wittman model is of particular significance to my own model since the valence advantage of a candidate is equated to the superior reputation of a college. Therefore, I enhance my model by assuming that one college has a superior reputation for academic quality.

In short, Groseclose concludes in his revised Calvert-Wittman model that an increased valence advantage actually causes a candidate to moderate his policy choices. As this advantaged candidate gravitates closer to the political middle, the opponent will actually gravitate further away from the center (Groseclose, 2001). Groseclose notes that these results may seem unintuitive; however, they make sense (Groseclose, 2001). He explains his results by asserting that the relative importance of the valence advantage actually increases as the advantaged
candidate moderates, since the actual relevancy of the policy itself is deemphasized (Groseclose, 2001). If there are two candidates in a campaign with relatively similar policy positions, the significance of any perceived advantage between the two is magnified. These results can provide valuable insight as to the theoretical results of my own model. If a particular college has a prestigious academic reputation, this may cause the school to “moderate” its position and to devote more resources to consumption amenities, rather than focusing on its strong suite. Therefore, if two schools are relatively similar in terms of its attributes (gravitating towards the political middle) then the valence advantage of a historic reputation becomes even more pertinent and important.

**Model and Assumptions: College Types and Preferences**

In terms of my own model, I have utilized Groseclose’s intuition in order to adapt his findings to my versions of candidates and voters. I have developed a simple interpretation of the preferences and assumptions of the two agents in my model, colleges (at least two) and prospective students.

More specifically, I consider two colleges in my model, college A and college B, who compete by choosing policy decisions denoted by $a$ and $b$. Their chosen policies, referring to the percentage of tuition revenue devoted to academic purposes, are placed along a unitary distribution of all possible policy decisions. They also compete through their valence characteristics, $\varepsilon_A$ and $\varepsilon_B$. Unlike the college’s preferences for policy, the college’s valence advantage is exogenous from the model. The advantage is known to both the college and the prospective students, but neither party can influence the valence advantage. Additionally, it is
also assumed that a greater valence advantage corresponds to a greater likelihood that the school will attract the student. In theory, colleges are free to choose whichever policy suites it best and a student exists to fill every possible policy position on the spectrum. For purposes of simplicity, we must assume that both colleges charge the same tuition to their students and they merely decide on an educational policy, defined in this case as the percentage of tuition revenues the college chooses to allocate to academic means rather than to amenities. The educational policy that the institution chooses can be deemed as its preference. The colleges, therefore, can only compete on policy positions and not on valence characteristics.

Not only do colleges have preferences for policy, but also for the number of students that attend the school. I assume that their utility for policy takes the form, \((1-\lambda) (\gamma_i - z_i)^2\), where \((1-\lambda)\) describes how much the college values its policy, \(z_i\) is the ideal policy position of candidate \(i\), and \(\gamma_i\) is the college’s chosen policy position. The assumptions about this equation imply that utility is maximized when \(\gamma = z\), but colleges may choose not to set them equal in order to lure in students to their college.

Next, it is important to consider the value colleges place on the number of students who attend their school. Since the schools value both the number of students who attend and the chosen educational policy, the utility function for a college presented below incorporates both factors.

\[
\text{Utility of College: } \lambda \cdot N - (1-\lambda) (\gamma_i - z_i)^2
\]
This equation represents the overall utility function for colleges with respect to both the number of students that attend and the policy that the college chooses. The college places respective weights around its preferences for students and policy. The $\lambda$ describes how much the college values the “N” number of students, while the $(1-\lambda)$ describes how much the college values its policy.

It is also important to note how my own model deviates from Groseclose’s model. In a presidential campaign, a candidate can lose his ability to institute a policy if he loses the election. However, a college cannot “lose” on policy. In other words, a college can always choose its educational policy because no actual election is taking place. Although the college can, in fact, “lose” on students if the prospects decide on their alternative choice. When $\lambda=1$, the college is completely student-motivated, and when $\lambda=0$, the college is completely policy-motivated.

**Model and Assumptions: Student Types and Preferences**

Along with those of colleges, the preferences and underlying assumptions of the second agent, prospective students, must also be evaluated. Referring back to Kevin Stange’s paper, it can be asserted that student preferences are heterogeneous. The infinite numbers of students we are considering also have their own unique preferences, specifically on both the educational policies and valence characteristics of colleges. In other words, the student distribution has individualized preferences over the tuition dollars spent on education and the dollars spent on elements of experience, as well as the college’s branding and reputation. Furthermore, it can be assumed that the greater the valence advantage of a particular college (in this case, an academic
reputation), the greater the number of students who want to attend the school. The utility function for each student is presented below.

$$\text{Utility of Student } i: \quad \epsilon_i - (L_i - V)^2$$

In this equation, the $\epsilon$ is representative of the valence advantage of the chosen college. The second term indicates the difference between the $L$, or the educational policy of the chosen school, and the $V$, or the ideal educational policy preference for an individual student across the uniform distribution on the policy spectrum. The heterogeneity of students is truly embodied by this $V$ variable, since every student has his or her own unique preference for policy. Colleges are unaware of the “Vs” of students, but they believe that the distribution of Vs along the spectrum is uniform from 0 to 1. I assume that a student exists who fills every ideal point on the spectrum. It is also important to note that the $L$ term represents the same policy that the $\gamma$ term represents in the college’s utility function. Therefore, a student’s utility can also be represented as the following.

$$\text{Utility of Student } i: \quad \epsilon_i - (\gamma_i - V)^2$$

Therefore, a student will choose college A over college B only if the following condition is met.

$$\epsilon_A - (a - V)^2 \geq \epsilon_B - (b - V)^2$$

From these initial equations and assumptions, it can be seen that students and colleges participate in a Stackelberg competition game. The students are the players and colleges compete to shape their ultimate decisions and to obtain a higher quantity of students than the other, while
also reaching as close as possible to their ideal policy. Colleges will advertise their brand or their offerings to students, while the prospects respond to these advertisements by choosing the school which maximizes their overall utility.

**Results**

For any choice of \( a \) and \( b \), there exists a point of indifference for a student along the spectrum. That is, there is a unique point, \( v \), that represents the ideal point of an indifferent student. For instance, if \( a \) is located to the left of \( b \), the student attends college A if \( a \leq v \), assuming there is not yet a difference in valence between the two colleges. The breakeven \( v \) can be calculated by utilizing the utility function for a student.

A student will be indifferent between college A and college B if the following condition is met:

\[
\varepsilon_A - (a - v)^2 = \varepsilon_B - (b - v)^2
\]

After solving for \( v \), the position of an indifferent student can be found through the equation:

\[
v = \varepsilon_B - \varepsilon_A - b^2 + a^2 / 2(a - b)
\]

Given this equation, all students with a \( v \) to the right of the breakeven \( v \) will attend college B, while all students to the left of the breakeven \( v \) attend college A (still holding that \( a < b \)). Therefore, this breakeven \( v \) allows the simple calculation of the percentage of students who go to each college.
**Claim 1:** Assuming that \( Z_a < Z_b \), the breakeven \( v \) increases with \( \varepsilon_A \), falls with \( \varepsilon_B \), and rises with both \( Z_a \) and \( Z_b \). This can be shown mathematically through the implicit function theorem.

**Proof 1:** Given, \( \varepsilon_A - (Z_a - v)^2 - \varepsilon_B + (Z_b - v)^2 \), the following partial derivatives can be attained:

\[
\frac{\partial v}{\partial \varepsilon_A} = -\frac{1}{2(Z_a - Z_b)} > 0, \text{ v increases to the right as } \varepsilon_A \text{ increases.}
\]

\[
\frac{\partial v}{\partial \varepsilon_B} = \frac{1}{2(Z_a - Z_b)} < 0, \text{ v decreases to the left as } \varepsilon_B \text{ increases.}
\]

\[
\frac{\partial v}{\partial Z_a} = Z_a - v / Z_a - Z_b > 0, \text{ v increases to the right as } Z_a \text{ increases.}
\]

\[
\frac{\partial v}{\partial Z_b} = Z_b - v / Z_a - Z_b > 0, \text{ v increases to the right as } Z_b \text{ increases.}
\]

**Logic 1:** These results provide us with valuable implications as to the number of students who attend to each college. More specifically, the partial derivatives of \( v \) with respect to the valence advantages reveal that more students will choose to go to the respective college as its exogenous valence advantage increases. Under the condition \( a < b \), the breakeven \( v \) will drift to the right along the spectrum as A’s valence increases, allowing college A to capture a greater share of students. Similarly, the breakeven \( v \) will drift further to the left if B’s valence increases, allowing college B to capture a greater share of students. Similarly, the breakeven \( v \) will also increase with increases in both colleges’ ideal policies. In other words, as either \( Z_A \) or \( Z_B \) become more extreme, the breakeven \( v \) will also become more extreme and move along the distribution. As \( Z_B \) becomes more extreme, the movement of the breakeven \( v \) implies that more students attend
college A and fewer students choose college B. These results provide important information for the model going forward.

A. **Boundary Results: Colleges Only Care about Policy (λ=0)**

Consider the case in which $\lambda=0$ for both colleges. In this environment, colleges only focus on policy decisions and disregard any attempt to maximize the $N$ number of students who attend their schools. Therefore, the utility function can be written as follows under this case, where $\gamma_i$ equals $a$ for college A and $b$ for college B. Given the utility equation, the highest value of $U$ possible is 0.

\[
\text{Utility of College}_i: - (\gamma_i - z_i)^2
\]

**Claim 1a:** Under the first situation when $\lambda=0$ and neither college possesses a valence advantage (therefore, $\varepsilon_A = \varepsilon_B$), the Nash equilibrium set of spending policies is $[a, b] = [Z_A, Z_B]$. The number of students that attend each college will then depend on the relative locations of the two colleges’ ideal spending policies. Namely, the college whose ideal spending policy is closer to the median student will attract more students.

**Proof 1a:** When $\lambda=0$ and $\varepsilon_A=\varepsilon_B$, then the objective of each college is simple: choose a spending policy that is as close as possible to its ideal spending policy. The optimal strategy of each college is therefore $a = Z_A$ and policy $b = Z_B$. 
**Logic 1a:** If a college does not care to consider the number of students that attend the school, it has no incentive to deviate away from its ideal policy. When neither college holds a valence advantage, the only factor which sets one apart from the other is its policy. More students decide to attend the college whose ideal spending policy is closer to the median student due to the fact that the college will receive half of the students, plus its share of students from the breakeven v. In other words, the breakeven v will lie between $a$ and $b$, so if $a$ is closer to the median than $b$, college A will receive 50% of the students plus half of the difference between its own policy and college B’s policy. College B, on the other hand, would receive 1 minus the percentage of students college A receives.

**Claim 2a:** Assume $\lambda=0$, $a < b$, but now $\epsilon_A > \epsilon_B$. college A now always holds a reputational advantage over B. The Nash equilibrium set of spending policies is still $[a,b]=[Z_A, Z_B]$. The same logic that is expressed in claim 1a also holds here. The number of students that attend each college will still depend on the relative locations of the two colleges’ ideal spending policies; however, more students will now attend college A due to the additional reputational advantage. In other words, the breakeven v moves further to the right because of the increased value of $\epsilon_A$. Even in the presence of this valence advantage, any deviation of policy from the median will result in fewer students for college A.
Proof 2a: When λ=0, but A now holds a valence as compared to B, the Nash equilibrium will not change. This is because each college still maximizes its utility when γ=z. However, more students will choose to attend A than in the situation without the valence. That is, the breakeven v will drift further to the right away from a along the distribution as the valence advantage of εₐ increases, allowing A to capture a greater share of prospective students. Even if B’s optimal choice was also 50%, A would still receive more students because of the extra valence advantage. Since the colleges do not care about the number of students who attend their schools when λ=0, the colleges will still choose their ideal policies, but A will be better off than it was before.

Logic 2a: Even when one college possesses a valence advantage over the other, both colleges will still choose a policy equal to their ideal. This is due to the fact that no weight is placed before the “N” term in the college’s utility function because the school only cares about policy. Even when A holds a valence advantage, the highest utility B can derive is when b=Zₜ. There is no incentive on the part of either school to deviate from their Zs.

*Theorem 1:* Therefore, under the extreme condition when λ=0, colleges should always choose their ideal policies. When college A holds an advantage of an historic reputation for academic quality over college B (that is, εₐ>εₜ), A will receive more students than B at the same Zs due to the valence advantage.
B. Boundary Results: Colleges Only Care about Numbers of Students ($\lambda=1$)

Under the opposite scenario when $\lambda=1$, colleges are only concerned about maximizing the number of students who attend their schools. Therefore, the colleges do not feel tethered at all by their ideal policies. The new utility function can be therefore written as follows:

\[
\text{Utility of College}_i = N
\]

From this function, it is evident that a college will maximize its utility by choosing a policy which maximizes the number of its students.

**Claim 1b:** Consider the case when $\lambda=1$ and $\varepsilon_A=\varepsilon_B$. If $a < .5$, then $b$ should choose a policy that equals $a +$ a miniscule amount. In other words, $B$ should choose a policy just to the right of the policy of $A$.

**Proof 1b:** When $\lambda=1$ and $\varepsilon_A=\varepsilon_B$, then the objective of each college is to maximize the number of students who enroll. Assuming that college $A$ has chosen $a < .5$, then college $B$ has three options: $b$ could be less than $a$, equal with $a$, or greater than $a$. If $b < a$, then college $B$ attracts fewer than 50% of all students. If $b=a$, then both colleges appear exactly equivalent to the students, and so it is assumed that half of all the students will go to each college. If college $B$ chooses $b > a$, then college $B$ gets all students whose ideal policies are greater than the breakeven value of $v$, while college $A$ attracts all the remaining students. Since the number of
students going to college B rises as \( b \) decreases, it is optimal for college B to choose a \( b \) that is as close as possible to \( a \), while still not being equal. This results in more than 50% of all students attending college B. Now, given these three options, we can analyze which choice is best for college B. Since \( a < .5 \), if \( b < a \), then \( U_B < .5 \). If \( b=a \), then \( U_B = .5 \) since the students would split their decisions between A and B. If \( b > a \) under the condition \( a < .5 \), then \( U_B \) would have to be greater than .5. As evidenced by these utilities, college B maximizes its utility when it chooses a spending policy \( b \) where \( b = a + \) a miniscule amount.

**Logic 1b:** This conclusion makes intuitive sense because when \( \lambda = 1 \), neither college cares about its policy. If A chooses a leftist policy, B’s strategic move would logically be to place itself just to the right of \( a \), in an effort to capture all of the students who value academic quality more than \( v = b \). When B places itself in this position, the school would maximize the number of students attending its school as much as possible considering the policy chosen by \( a \).

This situation is represented below: \( a \) is positioned further to the left of the median, so \( b \) will place itself just slightly to the right, capturing all of the excess \( v \), to right of \( a \) plus its original share of 50%.
**Claim 2b:** When $\lambda=1$ and $\varepsilon_A=\varepsilon_B$, if $a > 0.5$, then $b$ should choose a policy that equals $a$ - a miniscule amount. In other words, $B$ should choose a policy just to the left of the policy of $A$.

The proof and the logic are similar to the earlier case and so are suppressed.

**Claim 3b:** When $\lambda=1$ and $\varepsilon_A=\varepsilon_B$, if $a=0.5$, then $b=0.5$ as well. If college $B$ chooses any other policy, then the breakeven $v$ would be different than 0.5. College $A$ would then receive more than 50% of students and college $B$ would receive fewer than 50%, regardless of the direction of the policy (since $A$ would move directly next to $B$ in order to capture the greater share).

*Theorem 2:* When $\lambda=1$ and neither college $A$ nor college $B$ possesses a valence advantage (that is, $\varepsilon_A=\varepsilon_B$), then a Nash equilibrium always exists and is unique at $[a,b]=[0.5,0.5]$.
Claim 4b: If \( \lambda = 1 \) and \( \varepsilon_A \neq \varepsilon_B \), each school is still trying to maximize its utility by choosing the policy which will maximize the \( N \) number of students which attend each school. Consider the case when \( \varepsilon_A > \varepsilon_B \). Since college A has a valence advantage, its ideal spending choice would be to choose the same policy as B, rather than choosing an \( a \) which is less than or greater than \( b \).

Proof 4b: If A mimics B’s policy and chooses \( a = b \), then A will maximize its students in the presence of this advantage. We can see this more clearly when looking at the equation for the breakeven \( v \). A student would be indifferent between college A and B when:

\[
V = \varepsilon_B - \varepsilon_A - b^2 + a^2 / 2(a-b)
\]

The key to understanding this scenario is focusing on the denominator of the breakeven \( v \) equation. When \( a = b \), the denominator is 0; therefore, a breakeven \( v \) does not exist under these circumstances, or is equivalent to 1. This means that if A mimics the policy of B while possessing a valence advantage, then all students will attend college A when \( v = 1 \). Since B receives no students and all of the students choose A, college A’s utility is maximized since the school is receiving all possible students.

Logic 4b: In the case when one college actually has the valence advantage of a historic reputation and does not care about policy, then mimicking the spending decision of the competing college will always yield the greatest utility. Logically speaking, if two colleges have the same policy but one is renowned for its reputation, then students will always want to choose
the distinguished school. College B, the disadvantaged school, is always worse off in this situation.

**Claim 5b:** Consider the case when $\lambda=1$ and $\varepsilon_A \neq \varepsilon_B$, but $a \neq b$. If $\varepsilon_A > \varepsilon_B$, college B will desire to have a different policy from college A (either higher or lower depending on whether $a > 0.5$ or $a < 0.5$). If $a > b$, then a portion of students along the spectrum will still choose B, even in the presence of a valence advantage.

**Proof 5b:** Under the situation when $a > b$, there will be a large area of $v_s$ along the spectrum that still prefer B, even though A has an advantage. Even though the breakeven $v$ will gravitate away from $a$, college B will still get students who attend the school. This situation is represented below:
As evidenced from the diagram, the majority of students still attend B when $a > b$ due to the fact that A chose such an extreme policy position. Even though A still receives more students than it would in a situation without valence, college B is still better off and will have a higher utility than A. As $a$ moves closer to $b$ along the distribution, so does the breakeven $v$. In other words, as A chooses to moderate its policy choice, the college receives more students than when it chooses an extreme policy further from the median. A’s utility increases as $a$ moves to the left.

**Logic 5b:** This conclusion makes sense due to the fact that B’s policy choice is more moderate than that of A. Since A has chosen an extreme policy that is greater than $b$, A will capture a smaller portion of the unitary distribution of students. A breakeven $v$ exists which is less than 1, which is indicative of the fact that students will still choose college B over A due to their preferences. In this case, the exogenous valence advantage increases the utility of A by increasing its students, but not by a large amount.

*Theorem 3:* When $\lambda = 1$ and $\varepsilon_A \neq \varepsilon_B$, the advantaged school should always mimic the policy of the disadvantaged school. This action decreases the relative importance of the policy and emphasizes the presence of the valence advantage. The disadvantaged college, however, always wants a spending policy that is different from that of the advantaged school because only by being different does it have a chance to attract any students at all. These incentives together imply that no Nash equilibrium exists in pure strategy because for any particular combination of spending policies, one school would like to deviate.
A. Results: Colleges Care about Both Number of Students and Policy ($\lambda = .5$)

In reality, colleges will be assigned a lambda somewhere between the values of 0 and 1. Given this value, colleges will maximize their welfare. This means that schools value both the number of students who attend their institution and also the spending policy they choose to promote. Each college has its own unique lambda depending on its own unique preferences for academic spending, which will maximize its utility. Therefore, the goal in this section is to maximize college $A$ and college $B$’s utilities, relative to the policy the other school chooses. For a chosen policy of college $B$, college $A$ has a best response to maximize its utility. Similarly, college $B$ has a best response to maximize its utility for a chosen policy of college $A$. A Nash equilibrium exists in this scenario, where both schools will be happy with their chosen places and will not want to deviate.

In order to solve this model, I created a computer program in Matlab with the help of Professor Svec. This computer program allowed me to solve for the optimal value of $\lambda$ and the resulting utilities of both colleges for any given value of lambda, $\varepsilon_A$, $\varepsilon_B$, $Z_A$, $Z_B$, and $b$. For a complete summary of results for all cases of lambda, please refer to the chart in the appendix.

Claim 1c: Consider the case when $\lambda = .5$, $\varepsilon_A = \varepsilon_B$, $Z_A = .33$, and $Z_B = .66$. In this situation, each college values the number of students they receive and their spending policies equally, since $\lambda = .5$. A Nash Equilibrium here would be $a = .5$ and $b = .5$. In other words, college $A$ will have no incentive to deviate from the median if $B$ chooses a spending policy of .5.
**Proof 1c:** When $\lambda = .5$, assume to begin that A and B both place themselves at the median. If $a = .5$, then $U_A = .2356$. If $b = .5$ as well, then $U_B = .2372$. As $b$ moves to the right, closer to its ideal policy of .66, $U_B$ actually decreases due to the fact that B is sacrificing students in favor of policy. Similarly, as $b$ gravitates to the left, $U_B$ decreases for two reasons: B is moving further away from its ideal policy and is losing students. Therefore, college B has no incentive to leave the median, since deviation away from .5 would reduce its utility. Assuming $b = .5$, college A, similarly, has no incentive to deviate because more extreme policies to either the left or right of B would yield a lower utility than before.

**Logic 1c:** This conclusion makes intuitive sense because both colleges are essentially mirror opposites of each other. That is, the ideal points of A and B are equal in distance away from the median of .5. At $\lambda = .5$, $\varepsilon_A = \varepsilon_B$, the Nash is $[a, b] = [0.5, 0.5]$ because a deviation of either college towards its ideal point is more costly in terms of the loss of students than it is beneficial due to the gain from being closer to its ideal policy. This fact means that neither school would want to deviate from this Nash, even though both schools recognize that the result is one that has spending policies that are distant from their ideals. However, in the case when both colleges choose extremely radical policies, which are also equidistant from the median, the colleges will have more of an incentive to deviate. Under the same conditions as above, but $Z_A = .05$ and $Z_B = .95$, the colleges will deviate from the median and will choose $a = .3$ and $b = .7$. This movement makes sense due to the fact that both schools will still be the same distance away from the median, but they will be closer to their ideal policies and will have no damage to their utilities from loss of students.
**Theorem 4:** When $\lambda=.5$, $\varepsilon_A=\varepsilon_B$, and $Z_A$ and $Z_B$ are equidistant from the median on opposite sides (therefore, college A chooses a policy to the left and college B chooses a policy to the right), the Nash equilibrium will be $a=.5$ and $b=.5$. However, when the colleges have extremely radical Zs, such as .05 and .95, the schools have more of an incentive to deviate from the median since it will be beneficial to both parties, and will choose [.3,.7]. This solution is still mirror distance from the median on both ends.

It is now necessary to explore what happens to the Nash equilibrium when $Z_A$ and $Z_B$ are not mirror distances away from the median. I considered several cases where $Z_B$ switched values to illustrate how the Nash changes when a college alters its spending policy.

**Claim 3c:** When $\lambda=.5$, $\varepsilon_A=\varepsilon_B$, and $Z_A=.33$, college B’s ideal policy can either be more or less extreme relative to $Z_A$. When B’s ideal policy is on the extreme right of the spectrum, such as $Z_B=.9$, the Nash equilibrium moves infinitesimally, from $a=.5$ and $b=.5$ to $a=.5$ and $b=.501$.

**Proof 3c:** When both schools place their respective policies at the median (therefore, $a=b=.5$), college A’s utility $=.2356$ and college B’s utility $=.17$. If B chooses to move to the left of the median, the school is not only deviating further away from its $Z_B$, but it is also allowing A to capture the majority of students. This strategy proves to be even more hurtful to B’s overall utility than if B moves to the right, closer to its ideal. College B experiences a drastic decrease in
utility as it moves left, while A experiences a drastic increase in utility. As B moves to the right, towards its $Z_B$, $U_B$ also decreases, while $U_A$ continues to increase. This change in utility is indicative of the fact that the loss in students college B experiences from moving right is greater than the gain in utility the school achieves from moving closer to its ideal policy. Even though B is moving closer to its preferred policy, this movement allows A to capture a greater share of students, thus raising its own utility and simultaneously lowering college B’s utility. Even when B chooses its ideal policy ($b = Z_B = .9$), $U_B$ equals .13, which is significantly lower than the utility the school achieves at $b = .5$. However, the results indicate that both schools will be better off if $b = .501$, just slightly to the right of $b = .5$. In this scenario, A will still choose the median policy of .5, but B will be just slightly closer to its ideal policy. Therefore, the new Nash equilibrium exists at $a = .5$ and $b = .501$.

**Logic 3c:** Now, $[a,b]=[0.5,0.5]$ is not supportable as a Nash because a deviation to the right by B is beneficial, due to the fact that the costs associated with the loss of students is smaller than the gain associated with getting closer to its ideal policy. The reason for this holding now and not in the environment in claim 1c is because the squared term associated with the loss due to the policy being far away from its ideal is now relatively large, whereas this was not as large before when $Z_B = .66$. The larger discrepancy now between $b$ and $Z_B$ attributes to the slight movement of the Nash.

Logically speaking, when B’s ideal policy is more extreme, the school sacrifices all of the students who have more moderate or leftist preferences for policy if college B chooses its ideal. As B moves closer to its $Z_B$, the school allows A to capture a large share of students, since $Z_A$ is
relatively more moderate than $Z_B$. There is no reason for B to choose to move to the left of the median, since B would be both sacrificing students and moving further from its ideal. Only a slight movement to the right would raise B’s utility, but only minimally. Even though the increase in utility as B changes policy from $b=.5$ to $b=.501$ demonstrates to be only a slight improvement, these results are still significant because they are indicative of the direction a particular school will move in terms of its policy.

More specifically, these results occur due to the fact that $\lambda=.5$. When this is the case, colleges place a lot of value on the number of students they receive. Therefore, any action which increases the number of students that college A receives will be extremely hurtful to college B, even more so than large differences between its $b$ and its $Z_B$, or its actual and its ideal policies.

*Theorem 5:* When $\lambda=.5$, $\varepsilon_A=\varepsilon_B$, and $Z_A=.33$, only very extreme Zs (such as $Z_B=.86$) will cause the Nash equilibrium to move from $a=.5$ and $b=.5$. In the case of less extreme Zs closer to the median, the Nash will stay the same at $\{0.5, 0.5\}$.

**Claim 4c:** When $\lambda=.5$, $Z_A = .33$, $Z_B = .66$, but now $\varepsilon_A > \varepsilon_B + .3$, the implications for the model are now altered for when college A holds a prestigious reputation for academic quality over college B. When the valence advantage is relatively large (in this case, greater than .3) then the Nash equilibrium will be $a= Z_A$ and $b= Z_B$. 
**Proof 4c:** This claim proves to be true due to the fact that the valence advantage of a prestigious academic reputation in the eyes of a student outweighs the cost of the college’s spending policy being different from the student’s ideal policy. In other words, refer back to the utility function of a student attending college A. This student’s utility will be represented through the function $\varepsilon_A - (a - V)^2$. With a large $\varepsilon_A$ value, the student will always yield higher utility from attending college A than from attending college B, where the value of $\varepsilon_B$ would be significantly smaller. Therefore, in a situation with extremely high valence, all students will attend college A regardless of its position on the policy spectrum. Additionally, colleges will always go to their ideal point, because they have no incentive not to do so with such a high valence advantage.

**Logic 4c:** This claim makes sense because of the equation of the breakeven $v$. When the $\varepsilon_A$ term is large, the value for the breakeven $v$ will be greater than one. Therefore, regardless of the policy choices made by college A and college B, the breakeven $v$ value implies that all students will attend college A. Given this knowledge, both colleges choose their ideal points because this is where their utilities reach their maximum points. Since the high valence dwarfs any other potential factor for differentiation, college A will choose its ideal and college B will follow suit because it is already and will continue to lose on students.

**Claim 5c:** When $\lambda=.5$, $Z_A=.33$ and $Z_B=.66$, but now $0 < \varepsilon_A < \varepsilon_B + .3$, then $b = Z_B$ but $a$ will gravitate away from its $Z_A$ towards $Z_B$ as valence decreases. In other words, as college A’s
valence advantage diminishes, A’s policy will slide towards .66 in order to counteract this
decreasing valence, until $a$ eventually hits .66 and stops.

**Proof 5c:** Due to the fact that A’s valence advantage is now smaller than it was in claim 4c,
college A will choose to forego its ideal policy in order to mimic the policy of college B. By
mimicking the policy of college B, college A reduces the differences in spending policies
between A and B, making the colleges more and more similar. This act increases the relative
importance of the valence advantage when the colleges are essentially the same in policy. When
A mimics the policy of B, the breakeven $v$ calculates to be 1. Therefore, college A is able to
capture all of students in the spectrum when $a=b$ as a result of its valence advantage. The benefit
the college receives from all of the students attending A outweighs the cost endured by shifting
away from its $Z_A$.

**Logic 5c:** In this case when the valence advantage is relatively small, this movement to $Z_B$ on the
part of college A makes sense because the school wants to increase the importance of its valence
advantage. The historic reputation for academic quality becomes more important when the
colleges are not vastly different. Choosing .66 proves to be less hurtful for A than sacrificing
students to college B by reducing the breakeven $v$ to a value less than 1.

**Claim 6c:** When $\lambda=.5$ and $0<\epsilon_A<\epsilon_B+.3$, college B can actually alter its ideal spending policy
so as to put itself in a better position. In the case when $Z_A=.33$ and $Z_B=.9$, the extremeness of
college B’s policy actually allows the school to receive some students. The Nash equilibrium here would be \( a = .66 \) and \( b = Z_B \).

**Proof 6c:** When college B’s ideal policy is very extreme, the cost for college A of mimicking B’s policy is now larger than the benefit of receiving every student. Therefore, college A will choose to sacrifice some students to college B in order to maximize its utility. When A chooses .66, the breakeven v now has a value less than 1. The small damage done to college A’s utility from this slight loss in students is outweighed by the smaller difference between \( a \) and \( Z_A \) from choosing .66 as opposed to the \( Z_B \) of .9.

**Logic 6c:** These results not only make sense but they are also interesting in understanding the policy preferences of a disadvantaged school. Since college B is aware that college A has a valence advantage and will mimic its policy, a very extreme policy will dissuade college A from such imitation. Although college A has a valence advantage, it is not large enough for imitation under these circumstances. College A is only willing to spend so much on education over academic quality because at a certain point (.66), the costs of spending on education are too far from its ideal and too costly in terms of its utility. Therefore, college B is actually put in a better position if its ideal policy is further away from the median and more radical since the school will actually receive some students. A will not mimic this policy, even with the valence, because the policy preference of B is so distinct from its own.

Conversely, as \( \lambda \) decreases, the colleges would not choose the same set of policies. The logical prediction is that colleges will direct their policies more towards their ideal Zs, rather
than the policy which will allocate the most students to them. In this case, the Nash equilibrium will deviate from the median student more easily with less extreme Zs, due to the fact that less weight is placed on the number of students that attend the schools. As \( \lambda \) increases, however, it is predicted that colleges have even less of an incentive to deviate from the original Nash Equilibrium of \( a = .5 \) and \( b = .5 \), since the number of students who attend the schools become even more important. Therefore, colleges will be reluctant to deviate from the median towards their ideal Zs because of an increase in the breakeven \( v \). Such movements will cause reductions in student attendees.

**Key Points for When \( \lambda = .5 \)**

In the case when colleges place equal weight on students and on policy and have no exogenous historical reputation, they will be forced to spend their tuition dollars equally between academics and amenities if they want to maintain attendance at their respective school. If a school does have a radical preference for academic spending, even if it has no valence advantage, the school should only remotely move towards its ideal spending policy, since a larger deviation from the median would be hurtful to its utility. Only in the case of extremely high valence advantages should a school actually choose its ideal policy. When a school has a small valence advantage, its most strategic move would be to mimic the policy of its competitor. The disadvantaged competitor, in turn, stays true to its values and continues to choose its ideal policy. However, the competitor would be better off if its ideal policy was extreme.
B. **Results: Colleges Care about Policy More than Students ($\lambda = .3$)**

**Claim 1d:** When $\lambda = .3$, $\varepsilon_A = \varepsilon_B$, $Z_A = .33$, and $Z_B = .66$, the Nash equilibrium will no longer be $a = .5$ and $b = .5$ since the mathematical weight placed on students versus policy has now changed. More specifically, since college B’s utility function now reads as $(.3)N - (.7)(b - .66)^2$, it becomes evident that the college values how close it is to its ideal policy more so than the number of students that attend the school. Therefore, the schools will be more likely to drift away from the median and towards their ideal policies. The Nash equilibrium in this case will be $a = .437$ and $b = .55$.

**Proof 1d:** Under these conditions, college B derives utility of .1415 when placed at the median, while A places itself at $a = .437$, deriving utility of .1325. As opposed to the scenario when $\lambda = .5$, college A and college B both increase in their respective utilities as $b$ moves its policy along the spectrum to the right. However, A no longer wants to place its policy directly to the left of college B, since this action would distance itself too much from its ideal policy of $Z_A = .33$. As a result, college A’s utility continues to increase as B moves right, yet A refuses to move further to the right than $a = .437$. College B maximizes its utility when it reaches the point $b = .55$, and any additional movement towards its ideal will result in a reduction in utility. Thus, both colleges are happy at the points $a = .437$ and $b = .55$ due to the fact they are close enough to their ideal policies without sacrificing too many students.
**Logic 1d:** As predicted above, the policies of college A and B will diverge from the median and away from each other since the value of lambda has changed. This makes intuitive sense because both schools are less concerned with increasing the breakeven v, since they prefer policy over students. This preference for policy over students changes the Nash Equilibrium very drastically because the colleges are less reluctant to move towards their Zs in fear of losing too many students to the competing school.

**Claim 2d:** When $\lambda = .3$, $\varepsilon_A = \varepsilon_B$, $Z_A = .33$, and $Z_B = .9$, the Nash equilibrium will change more drastically since the weight placed on both students and policy has changed and college B has made its ideal policy more extreme. Therefore, the new Nash equilibrium will contain an $a$ and a $b$ which diverge away from each other, closer to their ideal Zs. The new Nash equilibrium in this scenario will be $a = .437$ and $b = .79$. The proof and the logic are subsumed under proof and logic 1d.

**Claim 3d:** In the case when $\lambda = .3$ but now $0 < \varepsilon_A < \varepsilon_B + .3$, the breakeven v moves to the right, as predicted by the implicit function theorem. In other words, as $\varepsilon_A$ increases, the breakeven v moves from $v = .7551$ (as in claim 2d) to $v = .8253$ due to the addition of a valence advantage. Therefore, the new Nash equilibrium will be at $a = .506$, $b = .87$ under the same conditions as above, when $\lambda = .3$, $Z_A = .33$, and $Z_B = .9$. 
**Proof 3d:** When $\lambda=.3$, colleges care less about the number of students who attend their school and more about their policy for spending. However, due to the addition of a valence advantage for college A, more students attend the school because of its historic reputation. This enables the breakeven $v$ to drift to the right of the spectrum towards the median, so that college A captures as many students as it can without severe hurt to its utility due to a large distance from its $Z_A$.

College A is able to move further from its ideal because of this valence, while college B moves closer to its ideal policy than it did in the situation without the valence advantage. Although college A’s new placement closer to the median is further from its ideal policy, it is not far enough away from its ideal in order to substantially hurt its utility. The benefit the school gets from the extra students that attend, coupled with the relatively close distance to its ideal policy, allows the school to maximize its utility at $a=.506$. Given this choice by college A, college B places itself at $b=.87$, close enough to its ideal but far enough so that it can still capture as many students as it can under these constraints.

**Logic 3d:** This finding is extremely interesting and intuitive due to the fact that it aligns nicely with the results in Groseclose’s paper. In his paper, Groseclose asserts that an advantaged candidate will gravitate closer to the median policy, while the disadvantaged candidate will diverge further away from the median. He explains his results by asserting that the relative importance of the valence advantage actually increases as the advantaged candidate moderates, since the actual relevancy of the policy itself is deemphasized (Groseclose, 2001). In terms of colleges, this makes intuitive sense because the disadvantaged school feels the need to differentiate itself through policy, as a result of the valence advantage held by college A. As
college A moderates, B becomes more extreme, but not extreme enough to the point where it will sacrifice all of its potential attendees.

**Key Points for When \( \lambda = .3 \)**

When colleges place less weight on the number of students who attend their school, they will be more likely to choose a spending policy closer to their ideal points. In other words, if a particular college wants to provide a large amount of amenities to their students, it can do so under these circumstances because it would be less hurtful to its utility. Similarly, extreme spending on academics over amenities would not be that harmful to the school’s well-being. When a valence characteristic is added, however, the school could moderate its policy without exerting harm on its overall well-being. The combination of a moderate policy and the valence advantage would allow the college to grasp the maximum amount of students possible, while staying relatively close to its ideal policy.

**C. Results: Colleges Care about Students More than Policy (\( \lambda = .7 \))**

**Claim 1:** When \( \lambda = .7 \), \( \varepsilon_A = \varepsilon_B \), \( Z_A = .33 \), and \( Z_B = .9 \), colleges will be even more reluctant to gravitate away from the median policy. This is due to the fact that colleges care a lot about students when \( \lambda = .7 \), so any transfer of student from college B to college A will be even more hurtful to B than in the situation when \( \lambda = .5 \). Therefore, the Nash Equilibrium will remain at \( a = .5 \) and \( b = .5 \).
**Proof 1e:** When \( a = b = .5 \), both colleges derive fairly similar utilities, at \( U_A = .3413 \) and \( U_B = .3020 \). When college B even slightly moves to the right, closer to its \( Z_B \) of .9, its utility is adversely affected. If B chooses \( b = .501 \), as opposed to \( b = .5 \), its utility will drop to .3019 while \( U_A \) increases to .3417. This is indicative of the fact that the school highly values students and desires to keep the breakeven \( v \) as small as possible. Even a movement as small as .001 closer to .9 leaves B worse off and makes college A better off. Therefore, neither college has the incentive to deviate from the median because the act of losing students would be so destructive to the school’s respective utility.

**Logic 1e:** Logically speaking, these results make sense for the opposite reasons of when \( \lambda = .3 \). In this case, colleges care more about the students they receive than the distance between their actual and ideal policies. Utility increases significantly as the N term in the college’s utility function rises. Therefore, movements towards \( Z_A \) and \( Z_B \) will be extremely hurtful to the colleges in terms of their respective utilities. Thus, the colleges will choose to converge to the center, where the breakeven \( v \) is the smallest and their utilities are maximized.

**Claim 2e:** In the case when \( \lambda = .7 \) but now \( 0 < \varepsilon_A < \varepsilon_B + .3 \), the Nash equilibrium will no longer be at the median due to the addition of the valence advantage. As proven in the implicit function theorem, the breakeven \( v \) will once again move to the right as \( \varepsilon_A \) increases. When \( Z_A = .33 \) and \( Z_B = .9 \), the new Nash equilibrium will be at \( a = .669, b = .9 \).
**Proof 2e:** When $\lambda=.7$, both colleges care about the number of students a lot more than they care about their policies. Therefore, given the valence advantage of A, college A will choose the policy position where the breakeven $v=1$. This means that each and every student along the spectrum attends college A. When $a=.669$, $v=1$. When $a=.668$, $v=.999$. Even a slight movement to the left towards its ideal policy will give college B some students; therefore, college A chooses .669. Given this choice by college A, college B knows it will receive no students. Therefore, it derives zero utility from the part of its utility function which considers students. The best utility B could possibly attain is equal to 0, since any deviation from its ideal policy at this point would further decrease its utility. Therefore, college B chooses its ideal policy, knowing college A will be receiving each student.

**Logic 2e:** When more weight is placed on students over policy, college A has no problem differentiating from its ideal policy. Rather, the most lucrative choice would be to choose the policy which would ensure that no students will attend the competition school. The cost of choosing a policy away from its ideal point is offset by the gain from getting every student. College B would merely be damaging its own utility further by choosing a policy which differs from its ideal. Therefore, the valence advantage puts college A in a position to capture each and every student when the weight is placed on students over policy.

**Key Points for When $\lambda = .7$**

When colleges place a large weight on the number of students who attend their school, they will be even more inclined to equalize their spending towards amenities and academics.
Even if a college heavily desires to spend their tuition dollars on academics, the cost of doing so would be extremely hurtful to its utility. As compared to the case when $\lambda = .5$, the same logic exists here but worse. When colleges feel pressure to attract as many students as possible, they should not spend more money on academics than on consumption amenities and student services. In the case when a school has a valence advantage, the advantaged school should choose the spending policy which ensures that all students will attend their school over the competitor. The disadvantaged school, on the other hand, must choose its ideal point in order to maximize its utility at 0. This is the best the disadvantaged school can do in the presence of the valence advantage.

**Closing Remarks and Discussion**

Even though these results exist in a theoretical world, they provide serious implications for the happenings within the market for higher education in the real world. In a situation where $\lambda = .5$, for example, colleges will be forced to equalize their spending between academics and amenities, even if the colleges have extreme ideal policies ($Z$s). In other words, colleges in this case may be forced to spend on amenities even if they have a strong preference for spending on academics. Even with a small valence advantage, a college will only be willing to spend so much on education before it becomes too costly to its ideal policy and its overall utility. However, based on the results, schools like MIT who have very extreme $Z$s and a presumably large valence advantage will break away from the Nash equilibrium of {.5, .5} because they are so heavily geared towards academics. MIT’s valence advantage is so large that it dwarfs any circumstance which may exert harm on its utility. Therefore, MIT is able to choose an extreme policy, like all
spending on education, if it so wishes because of the magnitude of its valence advantage. On the contrary, other schools which are more equivalent and ideologically less extreme will be forced to spend half of their money on academics and half on amenities if they wish to compete in the market. Schools like MIT will almost always win in a competition between itself and another school due to intensity of its historic reputation for academic quality. Other, less reputed schools will have to compete with current market tastes and trends.

However, when $\lambda$ equals a value other than .5, colleges will either feel more or less pressure to merge to the middle based on the weight they put on number of students versus spending policy. Based on the literature, it seems as though the value of $\lambda$ in the real world is at .5 or above for several reasons. First, the fact that tuition is becoming too expensive and prospective students are questioning the value of a college degree makes colleges fearful about their long-term solvency. Since the literature suggests that students have left the market due to rising tuition costs, colleges must attract and compete for the students who still remain in the higher education market. This suggestion hints to the theoretical assumption that the lambda value would be greater than .5, since colleges are about students a lot in their respective utility functions. Additionally, as Kevin Stange and Brian Jacob highlight in their paper, student preferences for “country club” attributes and amenities seem to be fairly universal across all college prospects (Jacob, 2013). Since colleges realize that students are extremely sensitive to the amenities offered at their particular schools and that they could attract prospects through this form of spending, it seems as though colleges are altering their ideal spending policies on the basis of this knowledge. Therefore, not only is it probable that their lambdas are greater than or equal to .5, but it is also probable that competing colleges are being forced to spend more on
consumption amenities than they otherwise would like in order to attract the students in the market.

Secondly, the fact that colleges are investing in large building projects and campus expansions, heavily geared towards nonacademic purposes, verifies the notion that colleges are trying to attract students through the channel of these consumption amenities. As stated in the 2016 report, “The State of Facilities in Higher Education,” 50 percent of recent campus growth is attributed to buildings used for nonacademic purposes (Gordian, 2016). This “amenities race” on the part of colleges exemplifies the competition that exists between institutions of higher education in the world today. If they do not equalize their spending and contribute to the pool of consumption amenities offered at their school, they will be crushed by current market forces. Similarly, the evidence suggests that colleges are actually cutting educational spending, i.e. through the channel of part time professors, in order to finance these large building projects and student services.

Therefore, by adapting the knowledge and intuition gained from the $\lambda=.5$ and $\lambda=.7$ cases, we can see that the advantaged college will alter its spending policy appropriately in order to grasp as many students as it can. Especially in the .7 case, the school with the historic reputation will choose the value which guarantees all students will attend its school, regardless of how far the policy is from its ideal. Realistically, this policy seems to be one heavily geared towards consumption amenities, which will provide a memorable experience for contemporary college students. Although these results exist in a theoretical world, they form a logical and a practical basis for understanding recent trends in higher education spending. In the future, this paper could be extended through an empirical analysis so as to test the accuracy of my model in the real world.
Appendix:

Results Summary

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*No Nash Equilibrium in Pure Strategy*
FRED CPI Growth for All Urban Consumers: Tuition, other school fees, and childcare from 1990-2015

“Consumer Price Index for All Urban Consumers: Tuition, Other School Fees, and Childcare.” *FRED*, 15 Nov. 2017, fred.stlouisfed.org/series/CUSR0000SEEB.
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